Comparative Analysis of the Guided Bomb Flight Control System for Different Initial Conditions

Marta Grzyb, Zbigniew Koruba
Kielce University of Technology, Al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce

Abstract: The article presents research on the influence of initial conditions on the self-guidance of a guided bomb towards a stationary ground target. The aim of the study was to investigate the correlation between the initial tilt angle of the guided bomb and the precision of impact, as well as the time required to reach the surface target. To fulfil this purpose, the flight control system of the guided bomb needed to be designed accordingly. The need to develop these systems arises from emerging information that Ukraine and Israel are converting unguided bombs into precision-guided ones. This justification is based on objective reasons. The article analyses the application of classical controllers. PI, PD and PID. Their task was to accurately guide a stationary ground target under various initial conditions. A preliminary method has been proposed for selecting optimal gain coefficients for the PID controller, which constitutes the main component of the autopilot of the guided bomb flight control system. A proprietary interpolation method was suggested, starting with the use of an optimization function in MATLAB software. The numerical findings are presented in a graphical manner.

Keywords: guided bomb, optimization, control system, initial conditions, PID controller, proportional navigation, MATLAB software, numerical simulation

1. Introduction

During the Vietnam War, the US Armed Forces confirmed the effectiveness of guided bombs, a type of precision weapon [1]. The ongoing armed conflict in Ukraine appears to confirm this, especially considering that 1159 guided aerial bombs were dropped on the country in September 2023. According to the Defense Express web portal, the utilization of this type of armament has doubled since May, when a record was set [2]. The armed conflict that started in early October 2023 suggests the use of guided bombs as precision weaponry. Israeli aircraft of the 5th generation F-35I Adir aircraft drop nearly 1-tonne GBU-31 JDAM guided bombs to provide direct support to ground troops. 110 kg bombs of lighter weight are used less frequently [3]. Armed conflict in the Gaza Strip also demonstrates another technique. Guided bombs are dropped no closer than 600 m from friendly forces, as per safety requirements. The utilization of air bombs of this type, dropped from a short distance, necessitates a high degree of precision. This precision can be guaranteed through the implementation of a well-designed guided bomb control system. It should be noted that guided bombs without propulsion have a lower risk of early detection and counteraction, even in the case of light guided bombs (Szakal I) [4].

This article presents research on the impact of the initial conditions of a guided bomb drop on the accuracy of hitting and the time of reaching the designated target for the designed control system. The analysis will cover three types of classical regulators. PI, PD and PID. Although controllers have been used for air-ground flying object systems such as missiles [5], UAVs [6], and rotorcraft [7], there is still a lack of research addressing guided bombs while taking into account the influence of initial conditions. The available literature indicates that several control methods have been used for guided bombs, such as a PD controller [8, 9], a hybrid controller [10], an optimal controller [11, 12], and a modified PID and PI-D controller [13].

The authors in the work [14] observe that the design of appropriate initial guidance conditions becomes an important problem that needs to be analysed. Research investigating the influence of initial conditions on the guidance process is scarce. They mainly pertain to the Impact-Time-Control Guidance (ITCG) methods [15]. The article [16] illustrates examinations conducted for various values of the slope angle while simultaneously preserving a constant impact duration. However, the studies presented in the work [17] analyse the impact of time errors for various initial course angle values, depending on the time constant of the autopilot dynamics model. The paper [18] suggests a two-stage control using the ITACG approach. The research concerned the verification of the effectiveness of the proposed guidance method, as well as its ability to adapt to different initial conditions and constraints for the final phase of flight. The purpose of using

Autor korespondujący:
Marta Grzyb, mgrzyb@tu.kielce.pl

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random initial conditions for a rocket projectile as presented in work [19] was intended to validate the effectiveness and practicality of the integrated guidance and control system. Similarly, the study [20] involved testing the guidance algorithms for three nominal trajectories that differed in the initial angle of projectile inclination. The guidance method proposed in [21] enables the calculation of impact times on the target, considering the initial conditions. Deviations of initial values from their nominal values may be interpreted as errors made by the pilot. And thus, the study [22] examined the effectiveness of the proposed approach and landing guidance law in relation to initial position errors for the unpowered reusable launch vehicle. Heading errors of –30 degrees and flight path angle errors of –10 degrees were considered. The guidance technique applied enabled the elimination of initial errors from the first flight phase.

Proportional navigation has been extensively studied over the past few decades [23, 24] and it is commonly used as a guidance method for various aerial objects, including guided missiles [25]. According to the authors [26], it is crucial to select a guidance method that can bring the guided bomb to the target with an error no greater than the warhead blast radius. The article assumes that a guided bomb's precision in hitting a ground target should have an error margin lower than 5 m.

The aim of this study was to analyse the influence of initial release conditions on the self-guidance accuracy for a guided bomb, as there is a lack of research on this topic in the available literature.

2. Geometrical and mass properties of a guided bomb

Geometrical and mass parameters of a guided bomb determine its physical dimensions that define its shape and size. Figure 1 shows the dimensions of the guided bomb under consideration. It is a rigid, axially symmetric solid with constant mass.

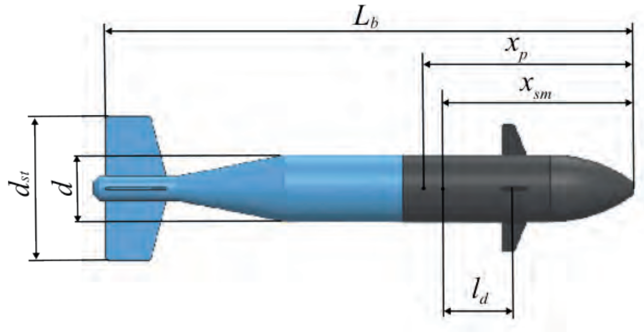

Fig. 1. Geometrical data of the guided bomb [27]
Rys. 1. Dane geometryczne bomby kierowanej [27]

Tables 1 and 2 present the main geometric parameters and mass-inertial data of the guided bomb, respectively.

The quantities described in tables 1 and 2 and presented in Fig. 1, can be defined as follows:

L_b – length of the guided bomb body;
d – diameter of the guided bomb body;
d_{st} – span of stabilisers of the guided bomb;
x_p – coordinates of the centre of pressure of the guided bomb;
x_{sm} – coordinates of the centre of mass of the guided bomb;
l_d – distance between the centre of pressure of the rudder and the centre of mass of the guided bomb;
m – guided bomb mass;
I_x – constant mass moment of inertia of guided bomb body in relation to x axis;
I_y – constant mass moment of inertia of guided bomb body in relation to y axis;
I_z – constant mass moment of inertia of guided bomb body in relation to z axis;
S_b – characteristic surface (cross-sectional area of the guided bomb).

3. Mathematical model of a guided bomb

The following assumptions have been made to analyse the movement of a guided self-seeking bomb towards a ground target:

- the guided bomb is a solid object with a fixed mass and inertial moments, as well as a stable positioning of the mass centre;
- the bomb's body is symmetrical along its axis;
- the Oxz plane is the geometric, mass, and aerodynamic symmetry plane;
- the guided bomb possesses three degrees of freedom.

The movement of the guided bomb requires the use of suitable coordinate systems to be described objectively and clearly. The article analyses the movement of a guided bomb solely in the vertical plane. That is due to the fact, that significant deviations of its flight parameters are visible for that plane. The equations of motion for the guided bomb were derived by taking into account Newton's law and the adopted coordinate systems. Figure 2 illustrates the forces that act on the guided bomb and the coordinate systems used.

Based on the given assumptions and coordinate systems, the non-linear equations of motion for the guided bomb being examined can be represented in the form [28]:

Table 1. Geometrical data of a guided bomb
Tabela 1. Dane geometryczne bomby kierowanej

L_b [m]	d [m]	d_{st} [m]	x_p [m]	x_{st} [m]	l_d [m]
0.846	0.1097	0.24	0.527	0.361	0.178

Table 2. Main mass-inertia data of a guided bomb
Tabela 2. Dane masowo-bezwładnościowe bomby kierowanej

m [kg]	I_x [kg · m^2]	I_y [kg · m^2]	I_z [kg · m^2]	S_b [m^2]
15.23	0.02503	1.0386	1.0386	0.00944

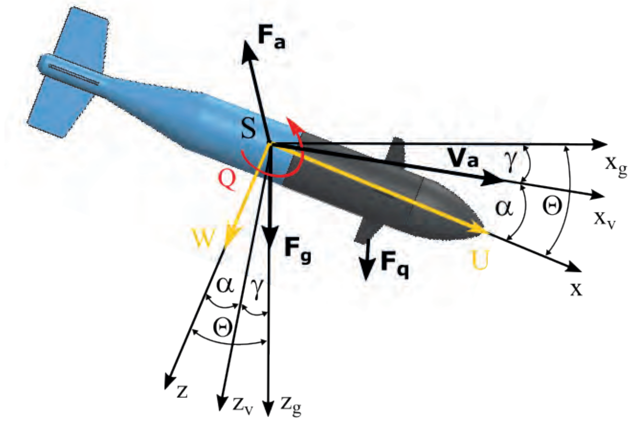

Fig. 2. Systems of force acting on a guided bomb, linear velocity and angular velocity [10]
Rys. 2. Układ sił działających na bombę kierowaną, prędkości liniowe oraz prędkości kątowe [10]

$$\dot{U} = \frac{F_x}{m} - QW \tag{1}$$

$$\dot{W} = \frac{F_z}{m} + QU \tag{2}$$

$$\dot{Q} = \frac{1}{I_y}\left[M_y + \left(I_z - I_x\right)PR\right] \tag{3}$$

$$\dot{\Theta} = Q \tag{4}$$

Figure 1 illustrates the forces acting on the guided bomb during its flight, including force of gravity $\boldsymbol{F}_g$, aerodynamic force $\boldsymbol{F}_a$, and control force $\boldsymbol{F}_q$. Equations (1)–(3) provide a formulation for the forces F_x and F_z, moment of force M_y exerted on the guided bomb during its flight [10]:

$$F_x = -mg\sin\Theta - C_{aX}\frac{\rho\left|\boldsymbol{V}_a\right|^2}{2}S_b \tag{5}$$

$$F_z = mg\cos\Theta + \frac{\rho\left|\boldsymbol{V}_a\right|^2}{2}S_b\left(-C_{aN}\left(\frac{W}{\left|\boldsymbol{V}_a\right|}\right) - C_{aNr}\left(\frac{Qd}{2\left|\boldsymbol{V}_a\right|}\right) - C_{Nd}\delta_w\right) \tag{6}$$

$$M_y = \frac{\rho\left|\boldsymbol{V}_a\right|^2}{2}S_b\left(dC_m\left(\frac{W}{\left|\boldsymbol{V}_a\right|}\right) + dC_q\left(\frac{Qd}{2\left|\boldsymbol{V}_a\right|}\right) - l_d C_{Nd}\delta_w\right) \tag{7}$$

where: g – acceleration of gravity, U, V – components of the velocity vector of the guided bomb in relation to the air in the boundary system Sxyz, Q – component of the angular velocity vector of the guided bomb body, ρ – air density, $\boldsymbol{V}_a$ – velocity vector of the guided centre of bomb mass in relation to the air, δ_w – deflection angle of the height rudder, C_{aX} – coefficient of the aerodynamic axial force, C_{aN} – coefficient of the aerodynamic normal force, C_{aNr} – coefficient of the aerodynamic damping force, C_m – coefficient of the aerodynamic tiling moment, C_q – coefficient of the damping tiling moment, C_{Nd} – coefficient of the aerodynamic control force.

4. Guided bomb control system

Selecting a control methodology is crucial for directing air-to-ground aerial vehicles, especially guided bombs. Compared to rocket projectiles, bomb lack propulsion, which makes the design of their control systems more intricate. It is crucial to recognise that the primary objective of guided bomb is to achieve precise impact on a ground target in the shortest time possible. We are referring to high-precision weapons. To meet these requirements, it is crucial to develop a dependable flight control system for guided bombs. The initial drop conditions of the guided bomb from the carrier must also be taken into consideration by this system. This is frequently neglected aspect in various analyses. No articles in the literature address the influence of the initial flight conditions (release conditions) of a guided bomb on the course of its flight and the effectiveness of the control system.

The controller id the main component of the autopilot and is responsible for generating control signals for the actuator system. The signals' values can be determined using various control methods, including classical PID control, sliding control, optimal control, or hybrid forms of controllers. Recently, guided bomb control systems have become more advanced, resulting in improved efficiency.

The work suggests using PI, PD, and PID controller algorithms for the control signal. For a proportional-integral-derivative (PID) controller, the signal is taken in the form described by [29]:

$$u_w = k_p e + k_i\int_{t_0}^{t_k} e\,dt + k_d\frac{de}{dt} \tag{8}$$

where: e – control deviation, k_p, k_i, k_d – constant gain coefficients of the PID controller.

In order to generate a control signal, the control deviation must be determined. In this case, the deviation can be expressed as:

$$e = \Theta_z - \Theta \tag{9}$$

where: Θ_z, Θ – set and current value of the pitch angle.

The inclination angle Θ_z value is a result of the chosen guidance method. Proportional navigation [30] is a frequently used method in the studies considered. The algorithm used to outline the adopted guidance method is described as follows:

$$\frac{d\gamma}{dt} = a_\varepsilon\frac{d\varepsilon}{dt} \tag{10}$$

Upon ejection of the guided bomb from the carrier, the inclination Θ_z angle aligns with the line of sight (LOS) inclination angle:

$$\Theta_z = \varepsilon \tag{11}$$

The main objective in analysing the flight control systems of a guided bomb is to accurately select the gain coefficients for the proposed PID controller. The following section outlines the method used the select them.

5. Simulation results

To analyse the effect of initial conditions on the guidance accuracy of a guided bomb on a ground target, simulation results were performed in MATLAB/Simulink software. Studies were carried out for three values of initial pitch angles: $\Theta_{b0} = \left\{+20°, 0°, -20°\right\}$, and for two values of initial velocity V_{b0} = {60 m/s, 120 m/s} for the guided bomb. For each of the cases considered, a numerical simulation was performed assuming the other initial condition values:

- initial position of the guided bomb: x_{g0} = 0 m, z_{g0} = 3000 m;
- initial position of the ground target: x_{c0} = 1000 m, z_{c0} = 0 m;
- initial ground target velocity: for a stationary target we assumed V_{c0} = 0 m/s;
- initial inclination angle of the flight path: $\gamma_0 = 0°$;
- initial angle of the attack: $\alpha_0 = 0°$.

The simulation studies aimed to verify the effectiveness of the proposed control system. The range of the guided bomb for the given initial conditions needed to be determined. This originated from the potential execution of the specified combat objective. Figure 3 presents the flight trajectories of the guided bomb for three tested values of the pitch angle $\Theta_{b0} = \left\{+20°, 0°, -20°\right\}$, and two initial flight velocity values of the guided bomb V_{b0} = {60 m/s, 120 m/s}.

It is assumed that the simulation studies used data on the parameters of the ground target obtained from the self-guided radar head, which accurately determines the parameters without any measurement errors. Before the carrier is launched, the guided bomb is provided with the coordinates of the ground target.

Guided bombs are characterised by their range, which increases with initial velocity and release angle, despite lacking their own propulsion. Table 3 shows the payload capacity of the guided bomb for three different pitch angles and two initial values of guided bomb flight speed.

Figure 4 shows a graphic representation of the method for dropping a guided bomb at a specific angle: $\Theta_{b01} = +20°$ we have a case of an ascending flight, $\Theta_{b02} = 0°$ – a case of a horizontal flight, and for $\Theta_{b03} = -20°$ – a case of a diving flight.

Guidance for the guided bomb was performed using proportional navigation algorithm with a_ε = 3.5 coefficient. In addition, for the control we are considering, it is assumed that the maximum deflection of the rudder $\delta_{\max} = \pm 20°$. The simulation step is set to be 0.01 s.

The initial conditions have been modified to replicate the most commonly encountered scenarios in the field of combat.

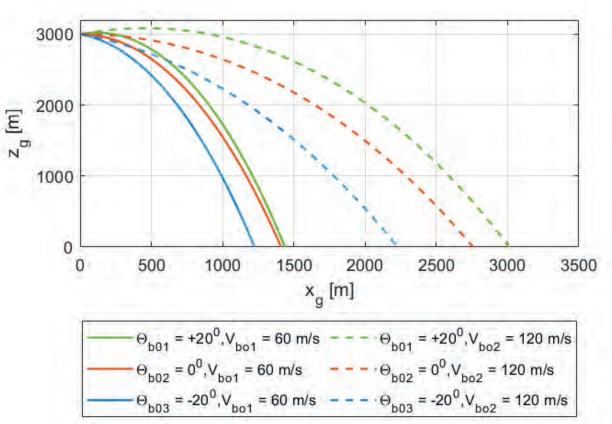

Fig. 3. Flight paths of a guided bomb for different initial conditions
Rys. 3. Tory lotu bomby kierowanej dla różnych warunków początkowych

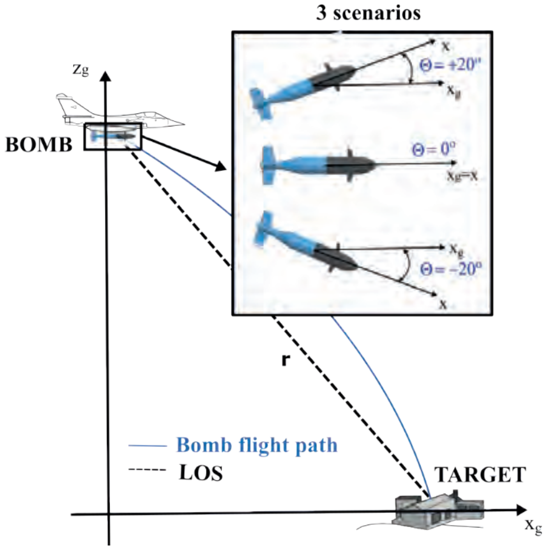

Fig. 4. Three scenarios for dropping a guided bomb homing at a ground target
Rys. 4. Trzy scenariusze zrzucenia bomby kierowanej na cel naziemny

Table 3. Payload capacity range for dive, horizontal, and climb maneuvers of the bomb
Tabela 3. Donośność bomby kierowanej dla lotu nurkowego, poziomego i wznoszącego

Initial pitch angle [deg]	Initial velocity [m/s]	Range [m]
$\Theta_{b01} = -20°$	$V_{b01} = 60$	1224.2
$\Theta_{b02} = 0°$	$V_{b01} = 60$	1410.8
$\Theta_{b03} = 20°$	$V_{b01} = 60$	1437.7
$\Theta_{b01} = -20°$	$V_{b02} = 120$	2229.0
$\Theta_{b02} = 0°$	$V_{b02} = 120$	2757.4
$\Theta_{b03} = 20°$	$V_{b02} = 120$	3012.0

5.1. Simulation results for the initial velocity of a guided bomb V_{b01} = 60 m/s

Simulation 1. The initial pitch angle is $\Theta_{b01} = +20°$

The first simulation assumed that the dropping of a guided bomb from the aircraft takes place at a pitch angle of $\Theta_{b01} = +20°$.

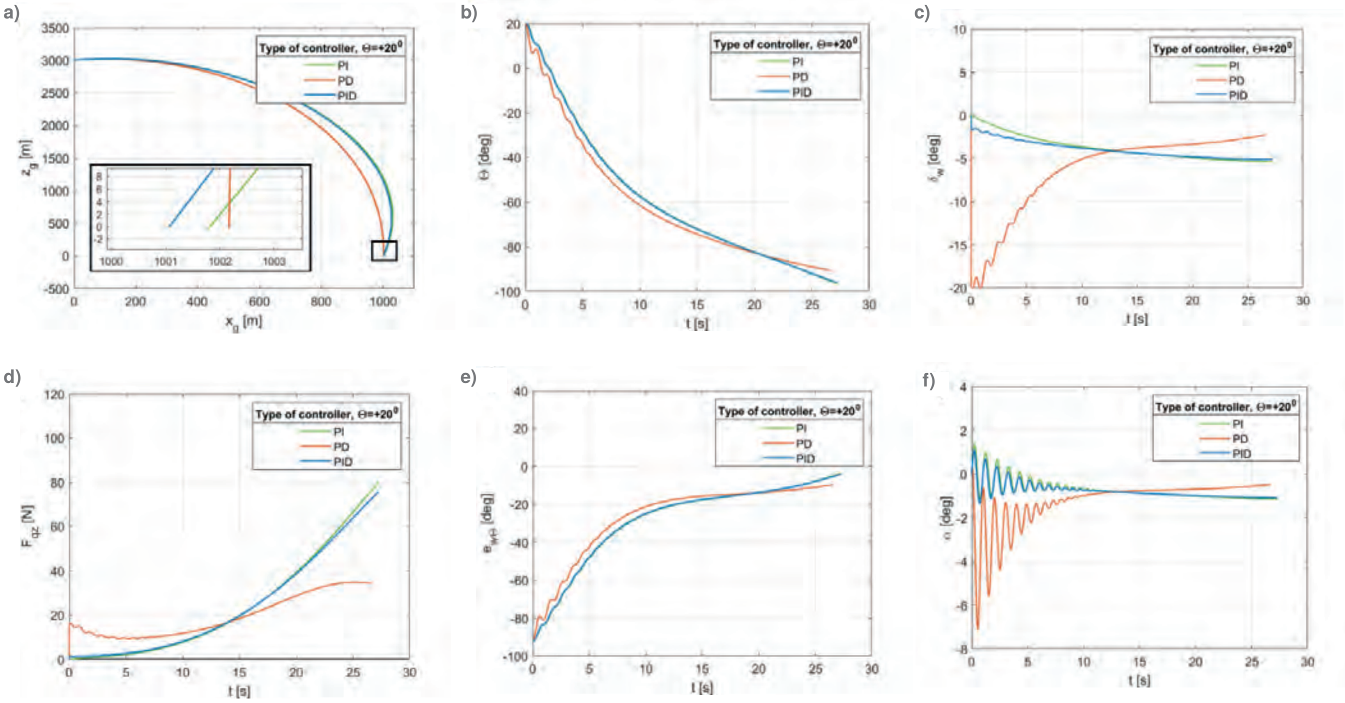

Fig. 5. Homing of a guided bomb on a stationary ground target for a pitch angle Θ_{b01} = +20°: a) trajectory, b) pitching angle, c) deflection angle of the height rudder, d) control force, e) angle deviations of bomb flight, f) angle of attack

Rys. 5. Naprowadzanie bomby kierowanej na nieruchomy cel naziemny dla kąta pochylenia Θ_{b01} = +20°: a) trajektoria, b) kąt pochylenia, c) kąt wychylenia steru wysokości, d) siła sterująca, e) kątowy uchyb sterowania, f) kąt natarcia

Simulation 2. The initial pitch angle is $\Theta_{b02} = 0°$

The second simulation assumed that the dropping of a guided bomb from the aircraft takes place at a pitch angle of $\Theta_{b02} = 0°$.

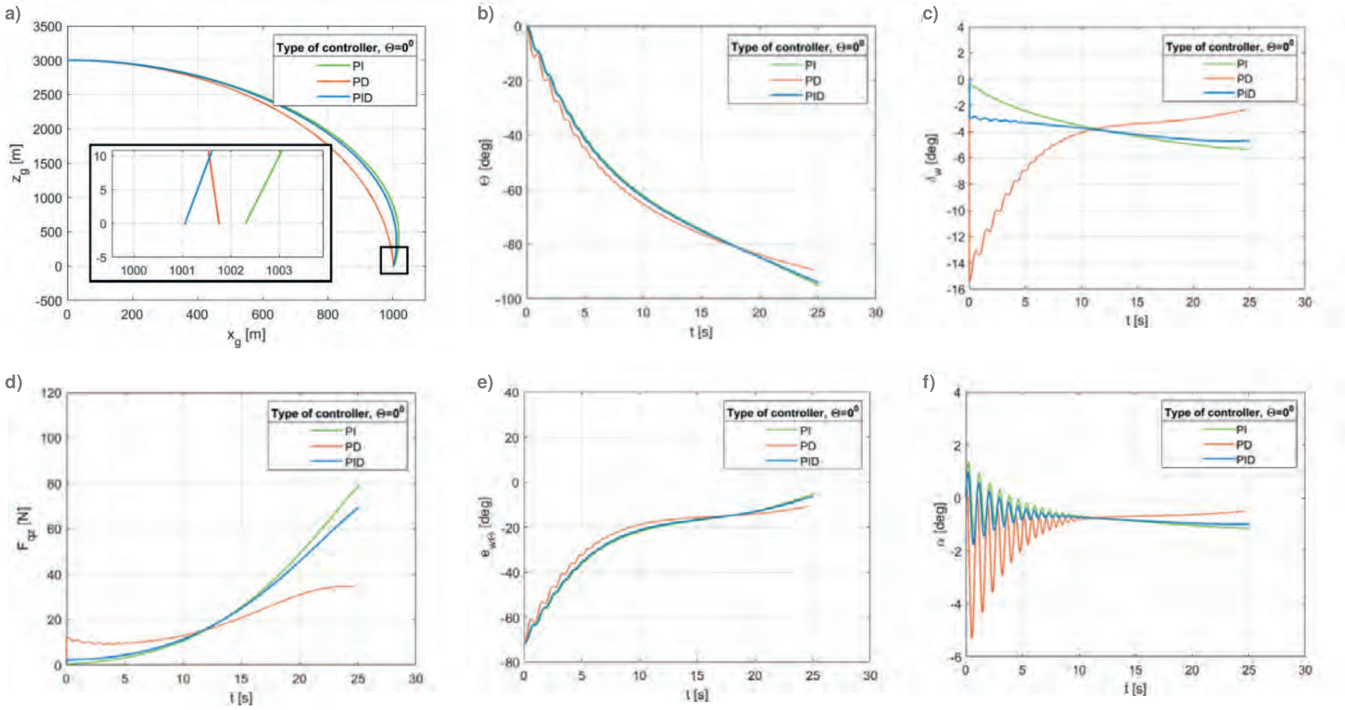

Fig. 6. Homing of a guided bomb on a stationary ground target for a pitch angle Θ_{b02} = 0°: a) trajectory, b) pitching angle, c) deflection angle of the height rudder, d) control force, e) angle deviations of bomb flight, f) angle of attack

Rys. 6. Naprowadzanie bomby kierowanej na nieruchomy cel naziemny dla kata pochylenia Θ_{b02} = 0°: a) trajektoria, b) kąt pochylenia, c) kąt wychylenia steru wysokości, d) siła sterująca, e) kątowy uchyb sterowania, f) kąt natarcia

Simulation 3. The initial pitch angle is $\Theta_{b03} = -20°$

The third simulation assumed that the dropping of a guided bomb from the aircraft takes place at a pitch angle of $\Theta_{b03} = -20°$.

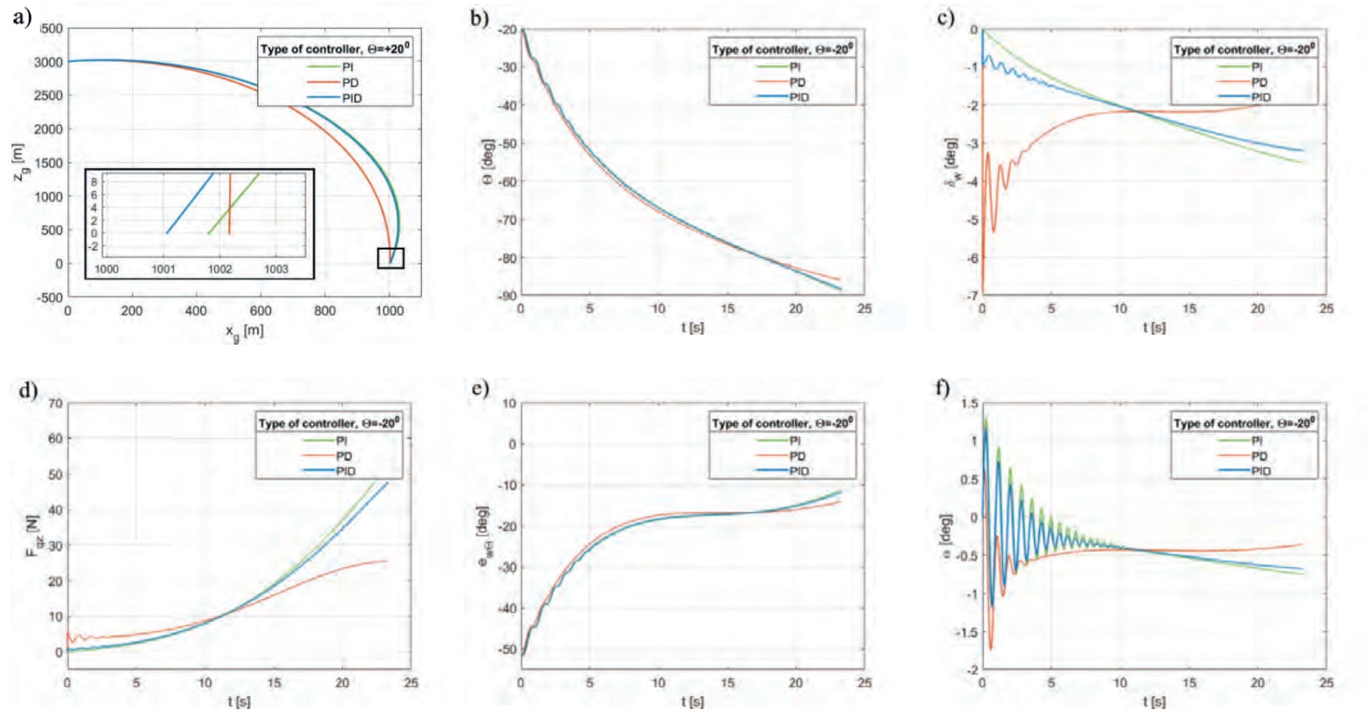

Fig. 7. Homing of a guided bomb on a stationary ground target for a pitch angle Θ_{b03} =–20° : a) trajectory, b) pitching angle, c) deflection angle of the height rudder, d) control force, e) angle deviations of bomb flight, f) angle of attack

Rys. 7. Naprowadzanie bomby kierowanej na nieruchomy cel naziemny dla kata pochylenia Θ_{b03} =–20° : a) trajektoria, b) kąt pochylenia, c) kąt wychylenia steru wysokości, d) siła sterująca, e) kątowy uchyb sterowania, f) kąt natarcia

Table 4. Summary of homing parameters for a guided bomb directed at a stationary target

Tabela 4. Zestawienie parametrów naprowadzania bomby kierowanej na nieruchomy cel naziemny

Type of controller	Guidance time [s]			Hit accuracy [m]		
	Θ_{b01}	Θ_{b02}	Θ_{b03}	Θ_{b01}	Θ_{b02}	Θ_{b03}
PI	27.29	25.13	23.36	1.95	2.47	2.93
PD	26.62	24.72	23.24	2.33	1.93	3.42
PID	27.24	25.05	23.34	1.23	1.22	1.52

Table 5. Summary of classic controller gain coefficient for a stationary target

Tabela 5. Zestawienie współczynników wzmocnień regulatora dla celu nieruchomego

Type of controller	Gain coefficients [s]		
	Θ_{b01}	Θ_{b02}	Θ_{b03}
PI	$k_p = 0.0001$ $k_i = 0.007$	$k_p = 0.005$ $k_i = 0.0086$	$k_p = 0.0001$ $k_i = 0.007$
PD	$k_p = 0.245$ $k_d = 0.045$	$k_p = 0.215$ $k_d = 0.004$	$k_p = 0.13$ $k_d = 0.19$
PID	$k_p = 0.018$ $k_i = 0.0066$ $k_d = 0.02$	$k_p = 0.042$ $k_i = 0.00725$ $k_d = 0.012$	$k_p = 0.018$ $k_i = 0.006$ $k_d = 0.02$

The simulations show that for all analyzed cases the assumed accuracy of hitting a ground target was achieved. The PD controller was characterized by the largest rudder deflection, even reaching its maximum $\delta_{\max} = -20°$ values. The use of the PID controller made it possible to reach the target with the smallest rudder deflection angle, which will minimize the energy consumption of the control actuation system. The angle of attack reached acceptable values. The time for a guided bomb to reach a ground target for all analyzed cases was similar. Of course, as expected, the smallest time was achieved for the angle $\Theta_{b03} = -20°$ (a case of a diving flight). Table 4 and 5 presents the values of the obtained parameters.

Table 5 displays the selected magnification ratios presents the selected gain coefficient for three classical controller structures: PI, PD and PID. The study focused on investigations for three initial values of the pitch angle: Θ_{b01}, Θ_{b02} and Θ_{b03}, as well as the initial velocity of V_{b01} = 60 m/s.

The simulations carried out show that the PD controller gives the maximum rudder deflection and the highest angle of attack. Table 4 presents the achieved results for the PID controller, showing the shortest guidance time and highest accuracy in hitting the ground target with the guided bomb.

5.2. Simulation results for the initial velocity of a guided bomb $V_{b02} = 120$ m/s

Another research group focused on the analysis of the impact of initial pitch angle values Θ_{b01}, Θ_{b02} and Θ_{b03} (as in chapter 5.1), but for a twice higher initial velocity of guided bomb flight.

Simulation 1. The initial pitch angle is $\Theta_{b01} = +20°$

The first simulation assumed that the dropping of a guided bomb from the aircraft takes place at a pitch angle of $\Theta_{b01} = +20°$.

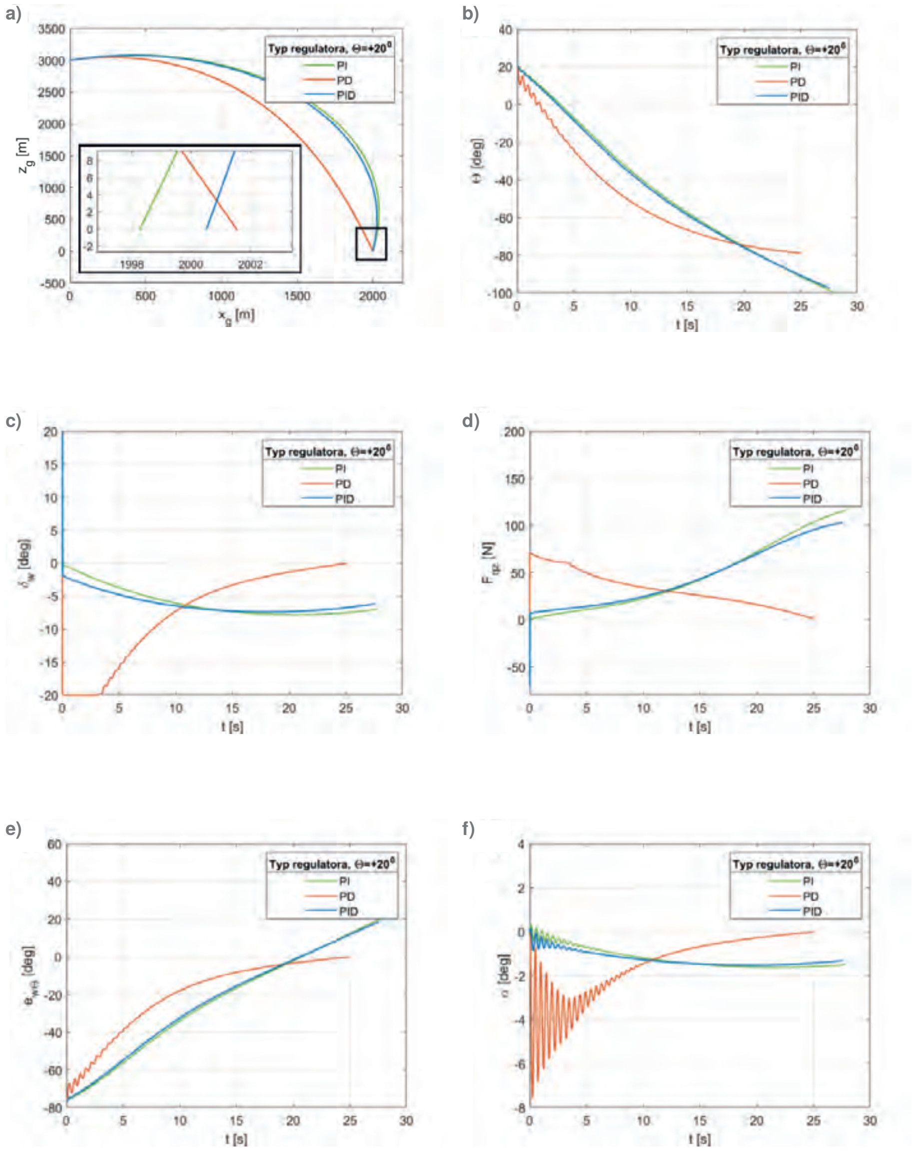

Fig. 8. Homing of a guided bomb on a stationary ground target for a pitch angle $\Theta_{b01} = +20°$: a) trajectory, b) pitching angle, c) deflection angle of the height rudder, d) control force, e) angle deviations of bomb flight, f) angle of attack

Rys. 8. Naprowadzanie bomby kierowanej na nieruchomy cel naziemny dla kata pochylenia $\Theta_{b01} = +20°$: a) trajektoria, b) kąt pochylenia, c) kąt wychylenia steru wysokości, d) siła sterująca, e) kątowy uchyb sterowania, f) kąt natarcia

Simulation 2. The initial pitch angle is $\Theta_{b02} = 0°$

The second simulation assumed that the dropping of a guided bomb from the aircraft takes place at a pitch angle of $\Theta_{b02} = 0°$.

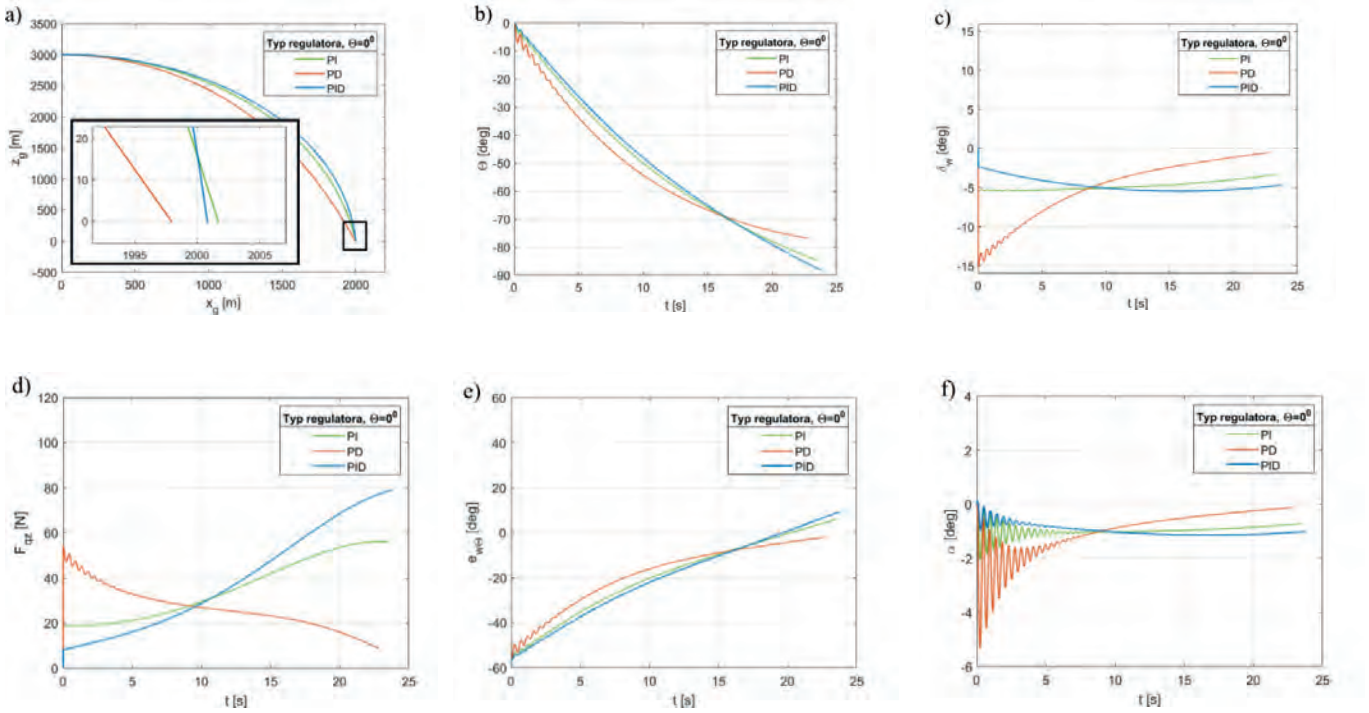

Fig. 9. Homing of a guided bomb on a stationary ground target for a pitch angle $\Theta_{b02} = 0°$: a) trajectory, b) pitching angle, c) deflection angle of the height rudder, d) control force, e) angle deviations of bomb flight, f) angle of attack

Rys. 9. Naprowadzanie bomby kierowanej na nieruchomy cel naziemny dla kata pochylenia $\Theta_{b02} = 0°$: a) trajektoria, b) kąt pochylenia, c) kąt wychylenia steru wysokości, d) siła sterująca, e) kątowy uchyb sterowania, f) kąt natarcia

Simulation 3. The initial pitch angle is $\Theta_{b03} = -20°$

The third simulation assumed that the dropping of a guided bomb from the aircraft takes place at a pitch angle of $\Theta_{b03} = -20°$.

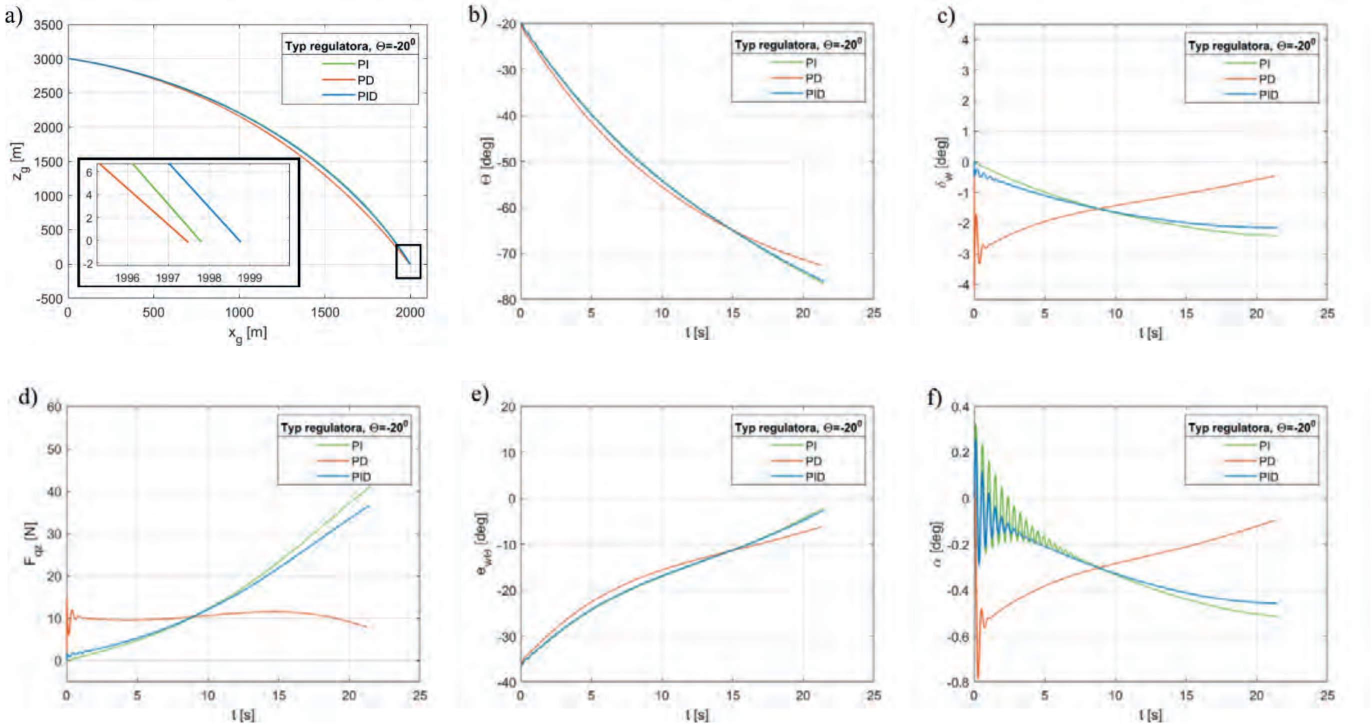

Fig. 10. Homing of a guided bomb on a stationary ground target for a pitch angle $\Theta_{b03} = -20°$: a) trajectory, b) pitching angle, c) deflection angle of the height rudder, d) control force, e) angle deviations of bomb flight, f) angle of attack

Rys. 10. Naprowadzanie bomby kierowanej na nieruchomy cel naziemny dla kata pochylenia $\Theta_{b03} = -20°$: a) trajektoria, b) kąt pochylenia, c) kąt wychylenia steru wysokości, d) siła sterująca, e) kątowy uchyb sterowania, f) kąt natarcia

Table 6. Summary of homing parameters for a guided bomb directed at a stationary target
Tabela 6. Zestawienie parametrów naprowadzania bomby kierowanej na nieruchomy cel naziemny

Type of controller	Guidance time [s]			Hit accuracy [m]		
	Θ_{b01}	Θ_{b02}	Θ_{b03}	Θ_{b01}	Θ_{b02}	Θ_{b03}
PI	27.87	23.49	21.46	1.58	1.87	2.03
PD	25.02	22.82	21.28	1.68	1.88	2.37
PID	27.55	23.67	21.44	0.66	0.93	1.08

Table 7. Summary of classic controller gain coefficient for a stationary target
Tabela 7. Zestawienie współczynników wzmocnień regulatora dla celu nieruchomego

Type of controller	Gain coefficients [s]		
	Θ_{b01}	Θ_{b02}	Θ_{b03}
PI	$k_p = 0.0025$ $k_i = 0.0107$	$k_p = 0.096$ $k_i = 0.0086$	$k_p = 0.00015$ $k_i = 0.00655$
PD	$k_p = 0.404$ $k_d = 0.002$	$k_p = 0.271$ $k_d = 0.009$	$k_p = 0.05$ $k_d = 0.225$
PID	$k_p = 0.026$ $k_i = 0.0104$ $k_d = 0.013$	$k_p = 0.0424$ $k_i = 0.011$ $k_d = 0.012$	$k_p = 0.0115$ $k_i = 0.0059$ $k_d = 0.034$

Figures 8–10 show research results for three values of initial angles and three controller structures. Increasing the value of the initial velocity to $V_{b02} = 120$ m/s primarily resulted in a shorted time for the guided bomb to reach the ground target. It can also be seen that with the higher velocity, we obtained a higher accuracy of the guided bomb hitting the target. The angle of attack for this case also takes acceptable values. Table 6 presents the key flight parameters of the guided bomb while autonomously targeting a stationary ground objective. Table 7 illustrates the gain coefficient for three controller structures: PI, PD, and PID. They have been carefully selected to achieve the highest accuracy in hitting the ground target.

6. The Procedure for Determining Gain Coefficients for PID Controller

The procedure of determining gain coefficients for each of the proposed classical controller structures is usually a highly time-consuming process that necessitates great precision. The literature shows numerous techniques for determining these factors for different types of study objects. Generally, these are approaches based on optimization theory [31, 32]. In the presented study, the *fmincon* function [33] was used in the MATLAB software. This function requires the input of several parameters, of which, after conducting a series of tests, the most difficult turned out to be determining the range of values for each of the sought-after gain coefficients and the initial value. Providing a considerable scope resulted in the absence of a solution. Consequently, utilizing this optimization feature necessitates a high level of expertise on the part of the investigator. A method for selecting gain coefficients k_p, k_i and k_d proposed in the article, which utilizes the interpolation method of individual gain coefficients depending on the pitch angle Θ. The research was conducted for two tested initial velocity values, namely: $V_{b01} = 60$ m/s and $V_{b02} = 120$ m/s. Due to the highest accuracy of guided bomb impact on the ground target, only PID controller was used for further research.

In order to design the function of changing PID k_p, k_i, k_d controller coefficients for the range of pitch angle $\Theta \in \langle -20, 20 \rangle$, a third-degree polynomial was used. Interpolation was carried out through the cubic spline function in MATLAB [34]. This type of interpolation function allows for obtaining a streamlined smooth curve [35], which has been defined using given data points.

Initially, the gain values for the pitch angle $\Theta_{b0} = \{-20°, 0°, 20°\}$ presented in Chapter 5, specifically in Table 5 and Table 7, were used.

Four arbitrary pitch angle values were selected for the purpose of verifying the proposed method: $\Theta_{b0} = \{-8°, -0.6°, 4°, 12°\}$, for which the values of the PID controller gain coefficients have been determined.

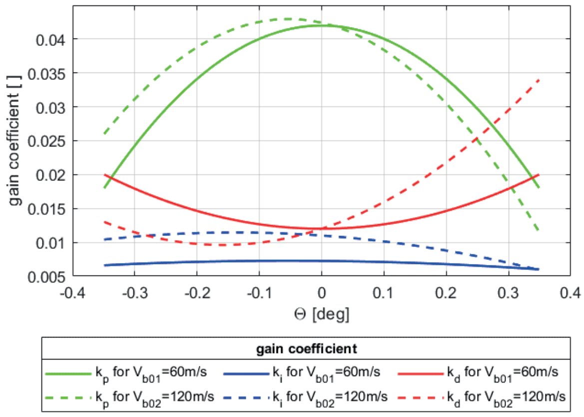

Fig. 11. Cubic spline interpolation for three data points
Rys. 11. Interpolacja funkcją wielomianową dla trzech punktów danych

In Fig. 11, the interpolation function of gain coefficients for the PID controller is presented for two velocity values, which utilized values for three data points.

Table 8 contains values of homing parameters, i.e. guidance time and accuracy of hitting the ground target, for two tested initial velocities and three data points.

In the next step, the number of data points was increased to five, and the initial values of the pitch angle were $\Theta_{b0} = \{-20°, -10°, 0°, 10°, 20°\}$. In Fig. 12, the interpolating function of gain coefficients for PID controller is presented for two velocity values and five data points.

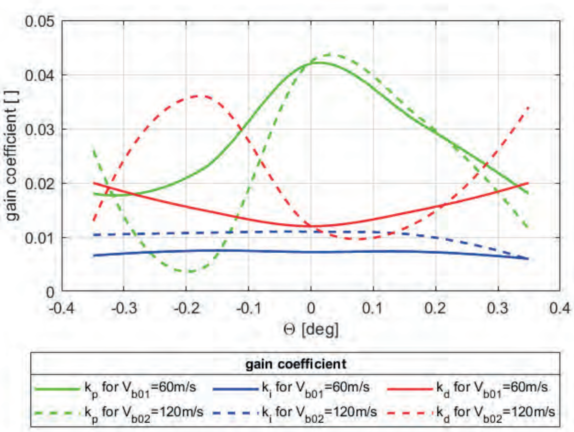

Fig. 12. Cubic spline interpolation for five data points
Rys. 12. Interpolacja funkcją wielomianową dla pięciu punktów danych

Table 9 contains values of homing parameters, i.e. guidance time and accuracy of hitting the ground target, for two tested initial velocities and five data points.

Based on the provided study findings, we can determine that even a minor quantity of data points has produced encouraging outcomes. Increase (even slight) in the number of these points has allowed to increase the effectiveness of PID controller operation. Consequently, it can be affirmed that this approach of selecting PID controller gain coefficients based on the initial change in the pitch angle value Θ_{b0} is suitable, albeit extremely time-consuming. To achieve this, the next stage of the work will consist of the automatic selection of gain factors for a classical controller using artificial neural network. The flight height of a guided bomb is taken into account together with the angle of pitch and velocity. It will allow for a quick selection of these coefficients using an onboard computer and the attainment of an efficient control system.

7. Conclusions

As indicated by the results of numerical investigations, the appropriate selection of initial conditions and gain coefficients of PI, PD, and PID controllers determines the effectiveness of the homing process of a guided bomb towards a ground target. This therefore requires the proper design of the flight control system for the guided bomb, implemented using classical controllers PI, PD or PID. The simulations conducted have shown that the most effective controller in terms of accurately hitting the ground target is PID. However, it is important to

Table 8. Overview of the homing parameters of a guided bomb attacking a fixed target
Tabela 8. Parametry naprowadzania bomby kierowanej atakującej nieruchomy cel naziemny

	3-Point Interpolation			
	$V_{b01} = 60$ m/s		$V_{b02} = 120$ m/s	
	Guidance time [s]	Hit accuracy [m]	Guidance time [s]	Hit accuracy [m]
$\Theta_2 = -8°$	24.30	6.08	22.57	66.94
$\Theta_3 = -0.6°$	24.99	1.04	23.69	6.36
$\Theta_4 = 4°$	25.45	3.85	24.51	24.51
$\Theta_1 = 12°$	26.32	18.51	26.23	166.59

Table 9. Overview of the homing parameters of a guided bomb attacking a fixed target
Tabela 9. Parametry naprowadzania bomby kierowanej atakującej nieruchomy cel naziemny

	5-Point Interpolation			
	$V_{b01} = 60$ m/s		$V_{b02} = 120$ m/s	
	Guidance time [s]	Hit accuracy [m]	Guidance time [s]	Hit accuracy [m]
$\Theta_2 = -8°$	24.33	2.71	22.89	17.62
$\Theta_3 = -0.6°$	24.99	0.44	23.71	0.33
$\Theta_4 = 4°$	25.45	1.41	24.43	13.19
$\Theta_1 = 12°$	26.33	6.74	26.09	99.83

choose appropriate controller coefficients for different bomb pitch angle. The precision of bomb homing will increase only if these conditions are met, ensuring no losses among accidental targets, especially civilian population. The recent military conflicts have demonstrated that this is not always the case.

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Analiza porównawcza systemu sterowania lotem bomby kierowanej dla różnych warunków początkowych

Streszczenie: W artykule przedstawiono wyniki badań określających wpływ warunków początkowych samonaprowadzania bomby kierowanej na nieruchomy cel naziemny. Skupiono się na analizie zależności początkowego kąta pochylenia bomby kierowanej na dokładność trafienia oraz czas potrzebny na dotarcie do celu naziemnego. W tym celu należało odpowiednio zaprojektować system sterowania lotem bomby kierowanej. Zasadność tych badań wynika między innymi z konieczności rozwoju tych systemów ze względu na pojawiające się informacje, że zarówno Ukraina, jak i Izrael przekształcają niekierowane bomby w ich precyzyjne odpowiedniki. W artykule poddano analizie zastosowanie regulatorów klasycznych: PI, PD oraz PID. Ich zadaniem było precyzyjne naprowadzanie na naziemny cel nieruchomy dla różnych warunków początkowych. Dodatkowo zaproponowana została wstępna metoda doboru optymalnych współczynników wzmocnień dla regulatora PID, stanowiącego główny element autopilota systemu sterowania lotem bomby kierowanej. Wyniki badań numerycznych zostały przedstawione w postaci graficznej.

Słowa kluczowe: bomba kierowana, optymalizacja, system sterowania, warunki początkowe, regulator PID, proporcjonalna nawigacja, oprogramowanie MATLAB, symulacja numeryczna

Marta Grzyb, PhD Eng.

mgrzyb@tu.kielce.pl
ORCID: 0000-0002-2258-6167

Graduate of the Faculty of Mechatronics and Mechanical Engineering at the Kielce University of Technology (2009). She is a member of the Machine Design department, which is headed by prof. Zbigniew Koruba. She received her PhD in technical sciences in 2023. She won a distinction for her doctoral thesis in the national competition „Young Innovators". She deals with the problems of guidance systems for air-to-ground flying objects, including primarily guided bombs. In addition, her interests concern modelling and analysis of the impact of atmospheric disturbances on the flight of such objects. She is the author of about 25 publications and co-author of six patents.

Zbigniew Koruba, DSc PhD Eng.

ksmzko@tu.kielce.pl
ORCID: 0000-0003-0715-593X

Graduate of the Mechanical Faculty of the Kiev Institute of Civil Aviation Engineers in Kiev. In 1982, he obtained the title of Master of Engineering, in 1990 the PhD degree, and in 2001 the post-doctoral degree of Technical Sciences. In 2010, he was awarded the title of Professor of Technical Sciences by the President of the Republic of Poland. Since 1984, he has worked at the Kielce University of Technology, holding successive positions of assistant, assistant professor, university professor and full professor. Since 2006, he has been the Head of the Department of Computer Techniques and Armament (currently the Department of Mechatronics and Armament). His research interests include problems of dynamics and control of gyroscopic systems, flying objects, in particular: homing short-range missiles, aerial guided bombs and unmanned aerial vehicles. Author/co-author of more than 300 publications on the above-mentioned topics, including 28 JCR-listed articles, five monographs, six textbooks and scripts. Promoter of eight closed doctoral theses. Reviewer of: nine proceedings for granting the academic title of professor and 19 for granting of the post-doctoral degree, 27 doctoral dissertations. President of the Polish Society of Theoretical and Applied Mechanics, Kielce Branch, member of the Kielce Scientific Society until 2014, expert on mechatronics in the State Accreditation Commission, member of the Dynamics of Systems Section of the Mechanics Committee of the Polish Academy of Sciences in 2003–2012. Member of numerous scientific committees of international, foreign and national conferences. Member of numerous scientific societies, committees and organizations, including PTMTS.