Analysis of mechatronic systems second class by the matrix method

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Abstract: In the paper an analysis of mechatronic systems by using matrix method has been described. On the base a real matrix method system is presented a model the member: electrics, electronics, mechanics, hydraulics and others in connections with feedback and without them has been examined. In the end an example at a control bus door for this purpose obtaining minimum time control has been presented.

Keywords: matrix method, mechatronics

1. Introduction

Investigating of dynamics in mechatronics systems which contain the members: electrics, electronics, mechanics, hydraulics, thermals, and others is important matter because the system has to be stable with regard for same parameters. In general, members of mechatronic systems are multipoles. In technical applications the system may be presented as two-port networks. The one is assumed as linear.

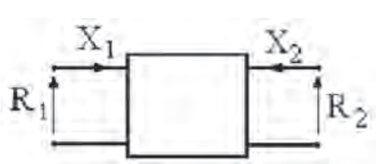


Fig. 1. A two-port network in general shape
Rys.1. Czwórnik w postaci ogólnej

It is meaning that $f(X_1, X_2, R_1, R_2)$ is linear function. A separate important problem is defining an amplitude range on surrounding at working point. The signals X_1, X_2, R_1, R_2 are Laplace or Fourier transform.

$$X = X(s)\,, R = R(s) \quad \text{or} \quad X = X(j\omega),\; R = R(j\omega) \qquad (1)$$

The two-port networks are described in form of differential or integrated equations. After Laplace (or Fourier) transformation the couple of linear equations have been got. In works on two-port networks are presented formulas between different forms of matrix. To consider the cascade connection of matrix has been got, as:

$$\begin{bmatrix} R_1 \\ X_1 \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} R_2 \\ -X_2 \end{bmatrix}; \quad \boldsymbol{A} = \cdot \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad (2)$$

2. Members of mechatronic systems and their connections

In the tab. 1 has been shown a quantity of mechatronic members.

With a progress of technique the new converters are being application, as for ex. ultrasonic, optics. In connection with it following mechatronic members may be presented:

– an electric-electronic member

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix} \qquad (3)$$

– a member as generator

$$\begin{bmatrix} \omega_1 \\ M_1 \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix} \qquad (4)$$

– a member as motor

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} \omega_2 \\ -M_2 \end{bmatrix} \qquad (5)$$

–

– a member as electromagnetic

–

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} V_2 \\ -F_2 \end{bmatrix} \qquad (6)$$

– a member as hydraulic (or pneumatic) converter

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} P_2 \\ -\vartheta_2 \end{bmatrix} \qquad (7)$$

– a member as thermal converter

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \boldsymbol{A} \cdot \begin{bmatrix} T_2 \\ -\phi_2 \end{bmatrix} \qquad (8)$$

Tab. 1. Quantity of mechatronic memebers
Tab. 1. Wielkości członów mechatronicznych

System	Electric	Pneumatic	Thermal	Mechanic	Mechanic (rotatable)
Potential R	Voltage U [V]	Pressure P [N/m^2]	Temperature T [K]	Velocity V [m/s]	Angular velocity ω [rd/s]
Flow X	Current I [A]	Flow (volume) V [m^3/s]	Flow (mass) V [kg/s]	Force F [N]	Moment M [Nm]

In connections of members the output signals at a previous member and input signals at a following member have to get the same physical character.

3. Input and output impedance of a member

Knowing a four-terminal member the impedance of members has been defined. Analogical to definition using in electrics

$$Z_{in} = \frac{U_1}{I_1} \quad \text{and} \quad Z_{out} = \frac{U_2}{I_2} \tag{9}$$

The definition has been extended for different mechatronics members

$$Z_{in} = \frac{R_1}{X_1} \quad \text{and} \quad Z_{out} = \frac{R_2}{X_2} \tag{10}$$

When the above values to present in frequency

$$Z_{in}(j\omega) = \frac{R_1(j\omega)}{X_1(j\omega)} \quad Z_{out}(j\omega) = \frac{R_2(j\omega)}{X_2(j\omega)} \tag{11}$$

Then, it may be calculation in frequency band of a work member.

a) A cascade connection of members

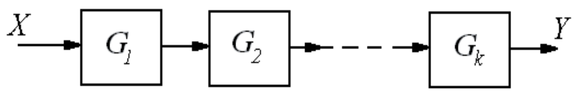


Fig. 2. Cascade connection of members
Rys. 2. Połączenie kaskadowe członów

The connection presented in the fig. 3 may be represented by transmittance

$$T = \frac{Y}{X} = G_1 \cdot G_2 \cdot \ldots \cdot G_k \tag{12}$$

When a following member do not load a previous member. Meaning, that

$$Z_{in_{k+1}}(j\omega) >> Z_{out_k}(j\omega) \tag{13}$$

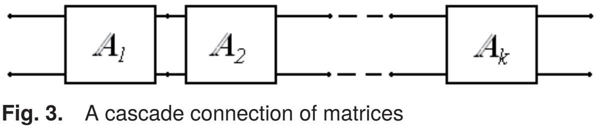


Fig. 3. A cascade connection of matrices
Rys. 3. Połączenie kaskadowe macierzy

Result matrix of the system is

$$\boldsymbol{A}_{res} = \boldsymbol{A}_1 \cdot \boldsymbol{A}_2 \cdot \ldots \cdot \boldsymbol{A}_k \tag{14}$$

If the condition (13) is not satisfy or impossible to estimation, then the matrix method should be applying in order to avoid a errors [7].

b) A system with feedback

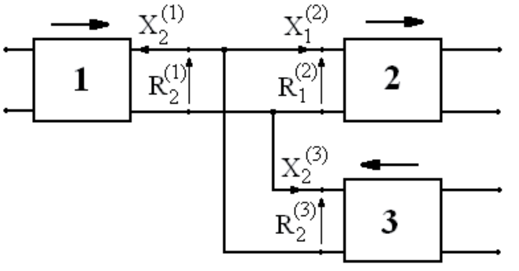


Fig. 4. A system with feedback at parallel. The arrows are meaning of signal at direction
Rys. 4. System ze sprzężeniem zwrotnym równoległym

$$\frac{R_2^{(1)}}{X_2^{(1)}} << \frac{R_2^{(2)}}{X_2^{(2)}}, \quad \frac{R_2^{(1)}}{X_2^{(1)}} << \frac{R_2^{(3)}}{X_2^{(3)}} \tag{15}$$

If the relation (15) is satisfied, then a block diagram may be presented as one-thread diagram.

c) The connection of parallel members

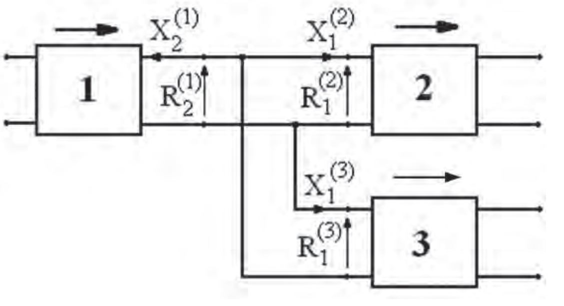


Fig. 5. The connection of parallel members
Rys. 5. Połączenie równoległe członów

The mutual loading should be satisfying the conditions

$$\frac{R_2^{(1)}}{X_2^{(1)}} << \frac{R_1^{(2)}}{X_1^{(2)}}, \quad \frac{R_2^{(1)}}{X_2^{(1)}} << \frac{R_1^{(3)}}{X_1^{(3)}} \tag{16}$$

If the relations (16) are satisfying then diagram may be presented in shape

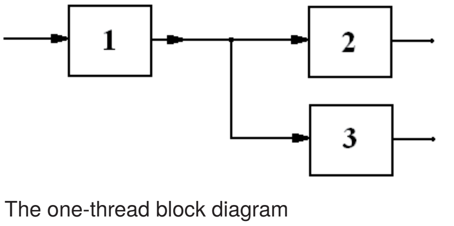


Fig. 6. The one-thread block diagram
Rys. 6. Jednonitkowy schemat blokowy

4. Matrix of systems with negative feedback

4.1. Connection with feedback of parallel-series

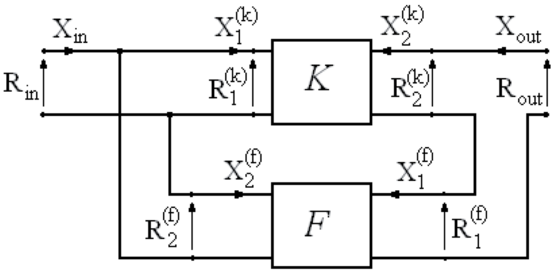


Fig. 7. A block's diagram with feedback of parallel-series
Rys. 7. Schemat blokowy ze sprzężeniem zwrotnym równoległo-szeregowym

Equations on input

$$X_{in} - X_1^{(k)} - X_2^{(f)} = 0 \tag{17}$$

where

$$X_1^{(k)} = X_{in} - X_2^{(f)} \tag{18}$$

and

$$R_{in} = R_1^{(k)} = R_2^{(f)} \tag{19}$$

It means negative feedback.
Output equations

$$X_{out} = X_2^{(k)} = X_1^{(f)} \tag{20}$$

and

$$-R_2^{(k)} + R_{out} - R_1^{(f)} \tag{21}$$

Now, the vector $[R_{out}, X_{in}]^t$ is

$$\begin{bmatrix} R_{out} \\ X_{in} \end{bmatrix} = \begin{bmatrix} R_2^{(k)} + R_1^{(f)} \\ X_1^{(k)} + X_2^{(f)} \end{bmatrix} = \begin{bmatrix} R_2^{(k)} \\ X_1^{(k)} \end{bmatrix} + \begin{bmatrix} R_1^{(f)} \\ X_2^{(f)} \end{bmatrix} \tag{22}$$

The vector's components in (22) having form

$$\begin{bmatrix} R_2^{(k)} \\ X_1^{(k)} \end{bmatrix} \overset{\text{det}}{=} \boldsymbol{D}^{(k)} \cdot \begin{bmatrix} X_2^{(k)} \\ R_1^{(k)} \end{bmatrix} \tag{23}$$

and

$$\begin{bmatrix} R_1^{(f)} \\ X_2^{(f)} \end{bmatrix} \overset{\text{det}}{=} \boldsymbol{H}^{(f)} \cdot \begin{bmatrix} X_2^{(f)} \\ R_2^{(f)} \end{bmatrix} \tag{24}$$

For instance a connection between D and $G = H$ is the following:

If $$\boldsymbol{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \quad \text{for} \quad \boldsymbol{D} = \begin{bmatrix} g_{22} & g_{21} \\ g_{12} & g_{11} \end{bmatrix} \tag{25}$$

Into consideration (23) and (24) in (22) we are having

$$\begin{bmatrix} R_{out} \\ X_{in} \end{bmatrix} = \left(\boldsymbol{D}^{(k)} + \boldsymbol{H}^{(f)}\right) \cdot \begin{bmatrix} X_{out} \\ R_{in} \end{bmatrix} \tag{26}$$

In the result of matrix

$$\boldsymbol{H}_{res} = \boldsymbol{D}^{(k)} + \boldsymbol{H}^{(f)} \tag{27}$$

H – type is as follows (24).

4.2. Connection with feedback at series-series

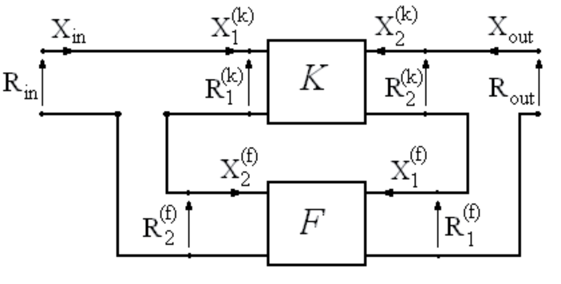


Fig. 8. A block's diagram with feedback of series-series
Rys. 8. Schemat blokowy ze sprzężeniem zwrotnym szeregowo-szeregowym

Equations on input

$$R_{in} - R_1^{(k)} - R_2^{(f)} = 0 \tag{28}$$

where

$$R_1^{(k)} = R_{in} - R_2^{(f)} \tag{29}$$

and

$$X_{in} = X_1^{(k)} = X_2^{(f)} \tag{30}$$

It means negative feedback.

Output equations

$$R_{out} - R_2^{(k)} - R_1^{(f)} = 0 \tag{31}$$

and

$$X_{out} = X_2^{(k)} = X_1^{(f)} \tag{32}$$

Now, the vector $[R_{out}, R_{in}]^t$ is

$$\begin{bmatrix} R_{out} \\ R_{in} \end{bmatrix} = \begin{bmatrix} R_1^{(k)} + R_2^{(f)} \\ R_2^{(k)} + R_1^{(f)} \end{bmatrix} = \begin{bmatrix} R_1^{(k)} \\ X_2^{(k)} \end{bmatrix} + \begin{bmatrix} R_2^{(f)} \\ R_1^{(f)} \end{bmatrix} \tag{33}$$

The vector's components in (33) having form

$$\begin{bmatrix} R_1^{(k)} \\ R_2^{(k)} \end{bmatrix} \overset{def}{=} \boldsymbol{Z}^{(k)} \cdot \begin{bmatrix} X_1^{(k)} \\ X_2^{(k)} \end{bmatrix} \tag{34}$$

and

$$\begin{bmatrix} R_2^{(f)} \\ R_1^{(f)} \end{bmatrix} \overset{def}{=} \boldsymbol{C}^{(f)} \cdot \begin{bmatrix} X_2^{(f)} \\ X_1^{(f)} \end{bmatrix} \tag{35}$$

For instance a connection between Z and C is the following:

$$\text{If} \quad \boldsymbol{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \quad \text{for} \quad \boldsymbol{C} = \begin{bmatrix} z_{22} & z_{21} \\ z_{12} & z_{11} \end{bmatrix} \tag{36}$$

Into consideration (34) and (35) in (33) we are having

$$\begin{bmatrix} R_{out} \\ R_{in} \end{bmatrix} = \left(\boldsymbol{Z}^{(k)} + \boldsymbol{C}^{(f)} \right) \cdot \begin{bmatrix} X_{out} \\ X_{in} \end{bmatrix} \tag{37}$$

In connection with it, the result matrix Z-type of system having form

$$\boldsymbol{Z}_{res} = \boldsymbol{Z}^{(k)} + \boldsymbol{C}^{(f)} \tag{38}$$

4.3. A connection with feedback of parallel-parallel

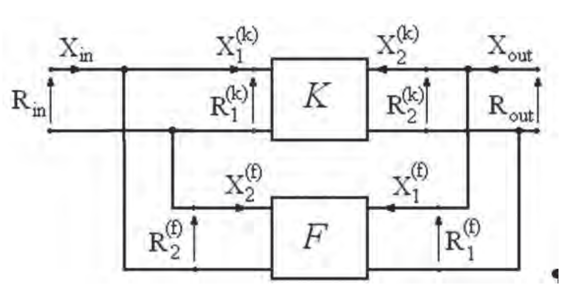


Fig. 9. A block's diagram with feedback of parallel-parallel

Rys. 9. Schemat blokowy ze sprzężeniem zwrotnym równoległo-równoległym

Equations on input

$$X_{in} = X_1^{(k)} + X_2^{(f)} \tag{39}$$

where

$$X_1^{(k)} = X_{in} - X_2^{(f)} \tag{40}$$

and

$$R_{in} = R_1^{(k)} = R_2^{(f)} \tag{41}$$

It means negative feedback.

Output equations

$$X_{out} = X_2^{(k)} + X_1^{(f)} \tag{42}$$

and

$$R_{out} = R_2^{(k)} = R_1^{(f)} \tag{43}$$

Now, the vector $\left[X_{in}, X_{out} \right]^t$ is calculated

$$\begin{bmatrix} X_{in} \\ X_{out} \end{bmatrix} = \begin{bmatrix} X_1^{(k)} + X_2^{(f)} \\ X_2^{(k)} + X_1^{(f)} \end{bmatrix} = \begin{bmatrix} X_1^{(k)} \\ X_2^{(k)} \end{bmatrix} + \begin{bmatrix} X_2^{(f)} \\ X_1^{(f)} \end{bmatrix} \tag{44}$$

The vector's components in (44) are having form

$$\begin{bmatrix} X_1^{(k)} \\ X_2^{(k)} \end{bmatrix} \overset{def}{=} \boldsymbol{Y}^{(k)} \cdot \begin{bmatrix} R_1^{(k)} \\ R_2^{(k)} \end{bmatrix} \tag{45}$$

and

$$\begin{bmatrix} X_2^{(f)} \\ X_1^{(f)} \end{bmatrix} \overset{def}{=} \boldsymbol{E}^{(f)} \cdot \begin{bmatrix} R_1^{(f)} \\ R_2^{(f)} \end{bmatrix} \tag{46}$$

For instance, a connection between Z and C is the following:

$$\text{If} \quad \boldsymbol{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \quad \text{for} \quad \boldsymbol{E} = \begin{bmatrix} y_{22} & y_{21} \\ y_{12} & y_{11} \end{bmatrix} \tag{47}$$

Then the expression (44) has a form

$$\begin{bmatrix} X_{in} \\ X_{out} \end{bmatrix} = \left(\boldsymbol{Y}^{(k)} + \boldsymbol{E}^{(f)} \right) \cdot \begin{bmatrix} R_{in} \\ R_{out} \end{bmatrix} \tag{48}$$

In connection with it, the result matrix Y-type of system is having a formula

$$\boldsymbol{Y}_{res} = \boldsymbol{Y}^{(k)} + \boldsymbol{E}^{(f)} \tag{49}$$

4.4. A connection with feedback of series-parallel

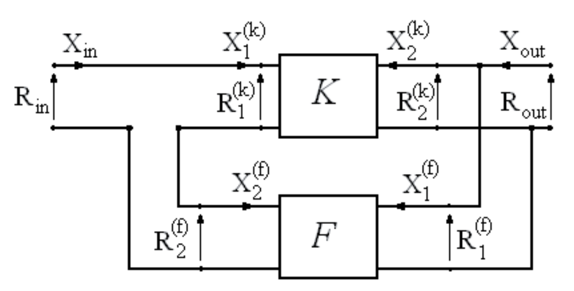


Fig. 10. A block's diagram with feedback of series-parallel

Rys. 10. Schemat blokowy ze sprzężeniem zwrotnym szeregowo-równoległym

Equations on input

$$R_{in} - R_1^{(k)} + R_2^{(f)} = 0 \tag{50}$$

where

$$R_1^{(k)} = R_{in} - R_2^{(f)} \tag{51}$$

and

$$X_{in} = X_1^{(k)} = X_2^{(f)} \tag{52}$$

It means negative feedback.

Output equations

$$R_{out} = R_2^{(k)} = R_1^{(f)} \tag{53}$$

and

$$X_{out} = X_2^{(k)} + X_1^{(f)} \tag{54}$$

Now, it will be calculated

$$\begin{bmatrix} R_{in} \\ X_{out} \end{bmatrix} = \begin{bmatrix} R_1^{(k)} + R_2^{(f)} \\ X_2^{(k)} + X_1^{(f)} \end{bmatrix} = \begin{bmatrix} R_1^{(k)} \\ X_2^{(k)} \end{bmatrix} + \begin{bmatrix} R_2^{(f)} \\ X_1^{(f)} \end{bmatrix} \tag{55}$$

It notice, that

$$\begin{bmatrix} R_1^{(k)} \\ X_2^{(f)} \end{bmatrix} \overset{def}{=} \boldsymbol{H}^{(k)} \cdot \begin{bmatrix} X_1^{(k)} \\ R_2^{(k)} \end{bmatrix} \tag{57}$$

and

$$\begin{bmatrix} R_2^{(f)} \\ X_1^{(f)} \end{bmatrix} \overset{def}{=} \boldsymbol{D}^{(f)} \cdot \begin{bmatrix} X_2^{(f)} \\ R_1^{(f)} \end{bmatrix} \tag{58}$$

For instance a connection between of components of matrix G and D is:

$$\text{If} \quad \boldsymbol{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \quad \text{for} \quad \boldsymbol{E} = \begin{bmatrix} g_{22} & g_{21} \\ g_{12} & g_{11} \end{bmatrix} \tag{59}$$

The expression (55) with regard to (57) and (58) having form

$$\begin{bmatrix} R_{in} \\ X_{out} \end{bmatrix} = \left(\boldsymbol{H}^{(k)} + \boldsymbol{D}^{(f)}\right) \cdot \begin{bmatrix} X_{in} \\ R_{out} \end{bmatrix} \tag{60}$$

In connection with it, the result matrix H-type of system is

$$\boldsymbol{H}_{res} = \boldsymbol{H}^{(k)} + \boldsymbol{D}^{(f)} \tag{61}$$

5. Concluding remarks

A presentation of systems in shape at a block diagram where members are two-port networks and describing by matrix is making possible a resultant matrix of system. By using at computer base of matrix transformation two-port networks the algorithm of calculation the matrix is quite simple.

6. Example

It should calculate a time constant at a integral circuit in feedback path at a control bus door like that a settling time will be minimum.

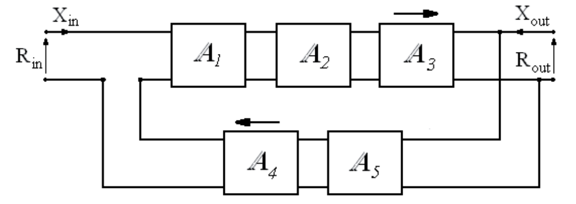


Fig. 11. A block diagram of matrix the control system door

Rys. 11. Schemat blokowy macierzy systemu sterowania drzwiami

A_1 – matrix of electronic amplifier
A_2 – matrix of power converter electric-hydraulic
A_3 – matrix of load
A_4 – matrix of shift-voltage converter
A_5 – matrix of integral circuit

The input parameters are voltage-current and output parameters are force and velocity. The scheme in fig. 11 may be reduction for the shape of fig. 12.

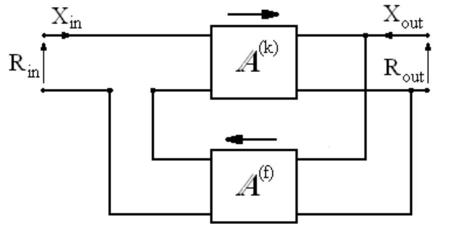


Fig. 12. A connection of feedback a series-parallel

Rys. 12. Macierzowy schemat blokowy ze sprzężeniem zwrotnym szeregowo-równoległym

in which

$$\left.\begin{aligned} A^{(k)} &= A_1 \cdot A_2 \cdot A_3 \\ A^{(f)} &= A_4 \cdot A_5 \end{aligned}\right\} \tag{62}$$

In the ex ample a response is X_{out} for unit step is $R_{out} = 1/s$. Using with the matrix H_{res} we have

$$T_{X_{out}R_{in}}(s) = \frac{1}{a_{11}(s)} \tag{63}$$

Where L(s), M(s) are polynomials with regard for s.

$$\text{If} \quad M(s) = s^2 + 2\alpha s + \omega_n^2 \tag{64}$$

Is a oscillation type, then settling time (with accuracy 2 %) getting

$$t_T = \frac{4}{a} \tag{65}$$

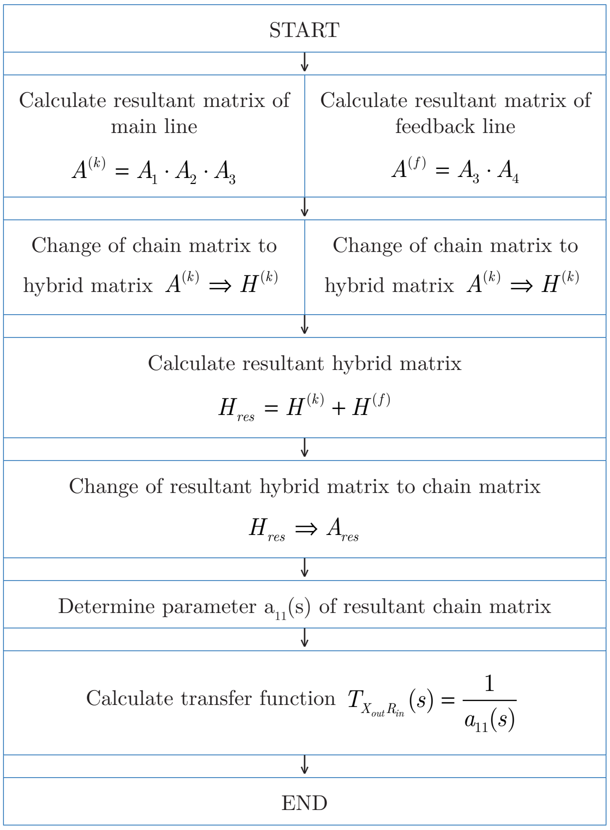
where a – coefficient by the s^1.

For higher order of systems the same formula is applied then is estimation.

In connection with that $a = f(T_c)$, T_c – time constant. It should minimize that value

$$\min f(T_c) \tag{66}$$

Algorithm of calculating T_c for presented system is the following.



7. Conclusion

In the paper the analyses of systems in which may be presented as two-port networks have been considered. To calculation the matrix method has been used. The conditions affording possibilities reduction at systems have been expressed.

The matrix of systems with feedback at different making possible to calculate a result matrix of systems in a quite simple. An example of control system algorithm has been presented in purpose a reckoning of a parameter that secure of minimum control time.

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Appendix 1 – two port's matrixes

Two port's matrixes are:

$$\left.\begin{array}{l}
\begin{bmatrix} R_1 \\ X_1 \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} R_2 \\ -X_2 \end{bmatrix}; \begin{bmatrix} R_2 \\ X_2 \end{bmatrix} = \boldsymbol{B} \begin{bmatrix} R_1 \\ -X_1 \end{bmatrix}; \boldsymbol{B} = \boldsymbol{A}^{-1} \\
\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = \boldsymbol{Z} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}; \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \boldsymbol{Y} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}; \boldsymbol{Y} = \boldsymbol{Z}^{-1} \\
\begin{bmatrix} R_1 \\ X_2 \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} X_2 \\ R_2 \end{bmatrix}; \begin{bmatrix} R_1 \\ X_2 \end{bmatrix} = \boldsymbol{G} \begin{bmatrix} X_2 \\ R_2 \end{bmatrix}; \boldsymbol{G} = \boldsymbol{H}^{-1}
\end{array}\right\} \tag{A1}$$

Define next original matrices

$$\begin{bmatrix} R_2 \\ R_1 \end{bmatrix} = \boldsymbol{C} \cdot \begin{bmatrix} X_2 \\ X_1 \end{bmatrix} \text{ if for ex. } \boldsymbol{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \quad \text{then } \boldsymbol{C} = \begin{bmatrix} z_{22} & z_{21} \\ z_{12} & z_{11} \end{bmatrix} \tag{A2}$$

$$\begin{bmatrix} R_2 \\ X_1 \end{bmatrix} = \boldsymbol{D} \cdot \begin{bmatrix} X_2 \\ R_1 \end{bmatrix} \text{ if for ex. } \boldsymbol{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \quad \text{then } \boldsymbol{D} = \begin{bmatrix} g_{22} & g_{21} \\ g_{12} & g_{11} \end{bmatrix} \tag{A3}$$

$$\begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \boldsymbol{E} \cdot \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \text{ if for ex. } \boldsymbol{Y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \quad \text{then } \boldsymbol{E} = \begin{bmatrix} y_{21} & y_{22} \\ y_{11} & y_{12} \end{bmatrix} \tag{A4}$$

and

$$\begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \boldsymbol{J} \cdot \begin{bmatrix} R_2 \\ R_1 \end{bmatrix} \text{ where } \boldsymbol{J} = \boldsymbol{C}^{-1} \tag{A5}$$

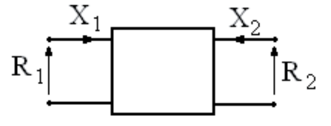
$$\begin{bmatrix} X_2 \\ R_1 \end{bmatrix} = \boldsymbol{L} \cdot \begin{bmatrix} R_2 \\ X_1 \end{bmatrix} \quad \text{where} \quad \boldsymbol{L} = \boldsymbol{D}^{-1} \tag{A6}$$

$$\begin{bmatrix} R_2 \\ R_1 \end{bmatrix} = \boldsymbol{M} \cdot \begin{bmatrix} X_2 \\ X_1 \end{bmatrix} \quad \text{where} \quad \boldsymbol{M} = \boldsymbol{E}^{-1} \tag{A7}$$

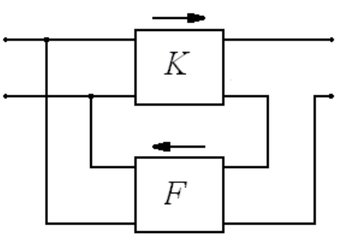
This type of matrixes are using in matrix systems with feedback.

Appendix 2 – two port's matrixes with negative feedback

Appendix 2 – Matrices of two-port networks with negative feedback



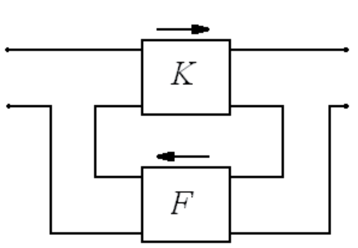
No. **1.** Kind of connection **parallel - series**



$$\boldsymbol{H}_{res} = \boldsymbol{H}^{(f)} + \boldsymbol{D}^{(k)}$$

$$\begin{bmatrix} R_1^{(f)} \\ X_2^{(f)} \end{bmatrix} = \boldsymbol{H}^{(f)} \cdot \begin{bmatrix} X_1^{(f)} \\ R_2^{(f)} \end{bmatrix} \quad \begin{bmatrix} R_2^{(k)} \\ X_1^{(k)} \end{bmatrix} = \boldsymbol{D}^{(k)} \cdot \begin{bmatrix} X_2^{(k)} \\ R_1^{(k)} \end{bmatrix}$$

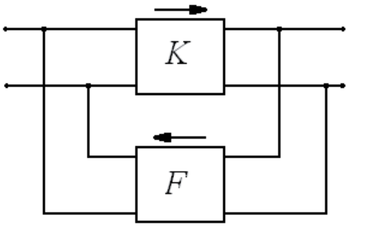
No. **2.** Kind of connection **series - series**



$$\boldsymbol{Z}_{res} = \boldsymbol{Z}^{(k)} + \boldsymbol{C}^{(f)}$$

$$\begin{bmatrix} R_1^{(k)} \\ R_2^{(k)} \end{bmatrix} = \boldsymbol{Z}^{(k)} \cdot \begin{bmatrix} X_1^{(k)} \\ X_2^{(k)} \end{bmatrix} \quad \begin{bmatrix} R_2^{(f)} \\ R_1^{(f)} \end{bmatrix} = \boldsymbol{C}^{(f)} \cdot \begin{bmatrix} X_2^{(f)} \\ X_1^{(f)} \end{bmatrix}$$

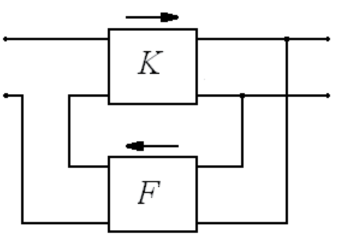
No. **3.** Kind of connection **parallel - parallel**



$$\boldsymbol{Y}_{res} = \boldsymbol{Y}^{(k)} + \boldsymbol{E}^{(f)}$$

$$\begin{bmatrix} X_1^{(k)} \\ X_2^{(k)} \end{bmatrix} = \boldsymbol{Y}^{(k)} \cdot \begin{bmatrix} R_1^{(k)} \\ R_2^{(k)} \end{bmatrix} \quad \begin{bmatrix} X_2^{(f)} \\ X_1^{(f)} \end{bmatrix} = \boldsymbol{E}^{(f)} \cdot \begin{bmatrix} R_1^{(f)} \\ R_2^{(f)} \end{bmatrix}$$

No. **4.** Kind of connection **series - parallel**



$$\boldsymbol{H}_{res} = \boldsymbol{H}^{(k)} + \boldsymbol{D}^{(f)}$$

$$\begin{bmatrix} R_1^{(k)} \\ X_2^{(k)} \end{bmatrix} = \boldsymbol{H}^{(k)} \cdot \begin{bmatrix} X_1^{(k)} \\ R_2^{(k)} \end{bmatrix} \quad \begin{bmatrix} R_2^{(f)} \\ X_1^{(f)} \end{bmatrix} = \boldsymbol{D}^{(f)} \cdot \begin{bmatrix} X_2^{(f)} \\ R_1^{(f)} \end{bmatrix}$$

Macierzowa analiza systemów mechatronicznych drugiego rzędu

Streszczenie: W pracy opisano analizę układów mechatronicznych drugiego rzędu za pomocą metody macierzowej. Wyznaczono oryginalne macierze wypadkowe członów o różnych połączeniach z ujemnym sprzężeniem zwrotnym. Na podstawie realnego systemu mechatronicznego, systemu sterowania drzwiami autobusu, wyznaczono minimalny czas zamykania drzwi.

Słowa kluczowe: metody macierzowe, teoria systemów, sprzężenie zwrotne, mechatronika

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