

Classification of FOPID and PID Controllers According to Input Signal Type: A 3D Crane Simulation Study

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Abstract: A 3D crane is a system used to transport heavy loads within a defined workspace. Five regulators are employed for its control. Three controllers regulate the position along the X, Y, and Z axes, while the remaining two use deviation information represented by the alpha and beta angle values. The aim of this paper is to conduct simulation studies to verify the feasibility of controlling a 3D crane system using FOPID controllers. Three control configurations were developed, differing in the number and placement of PID and FOPID controllers. Furthermore, the applicability of this control approach in real-time systems was investigated. Therefore, computation times were measured for the configuration consisting exclusively of FOPID controllers.

Keywordse: 3D crane, FOPID, PID, calculation time, real time requirements

1. Introduction

3D cranes and tower cranes are widely used in industry, construction, and transportation [3, 5, 6, 13]. They enable the movement of large objects. However, given their size, they must be precise. Containers on ships must be stacked close together to accommodate as many as possible. Also on construction sites, especially with large structures, a slight error in the positioning of large panels can cause a defect. Additionally, objects must be moved quickly, but this causes oscillations that must be minimized.

Fractional calculus is an extension of integer-order calculus [4]. The most important assumption is that the derivative does not need to be of integer order. This, of course, requires a significant amount of computational power. Additional information on fractional calculus can be found in the book [8].

A very popular benchmark in research is the PID controller [1, 2]. This is primarily due to its reliability and ease of use. Furthermore, it is frequently used in real-world systems.

The following chapters present a brief theoretical introduction, including the FOPID and the Grey Wolf Optimizer, which was used to determine the controllers coefficients. The next section describes mathematical model of the 3D crane and the implementation of the simulation of the entire system. The following chapter presents the achieved results, along with the execution times of the FOPID controllers. Article concludes with a brief summary.

2. Fractional Order PID controller

A generalization of the PID controller is FOPID, which uses fractional order operations. As a result, two additional variables appear in the transfer function. The formula is presented below [12]:

$$G_{\text{FOPID}} = K_P + \frac{K_I}{s^\lambda} + K_D s^\mu, \quad (1)$$

where K_P , K_I , K_D do not change with respect to the PID, meanwhile λ is the order of the integral and μ is the order of the differentiation.

In order to calculate the fractional derivative, the Oustaloup Recursive Approximation method was used, which can be described by the formula in the $\omega \in [\omega_b, \omega_h]$ [9]:

$$s^\lambda \approx C_0 \prod_{k=-N}^N \frac{s + \omega'_k}{s + \omega_k}, \quad (2)$$

where:

$$\omega'_k = \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+N+0.5(1-r)}{2N+1}}, \quad (3)$$

$$\omega_k = \omega_b \left(\frac{\omega_h}{\omega_b} \right)^{\frac{k+N+0.5(1+r)}{2N+1}},$$

$$C_0 = \left(\frac{\sqrt{\omega_h \omega_b}}{\omega_h} \right)^r \prod_{k=-N}^N \frac{\omega_k}{\omega'_k}$$

and also N is the order of approximation, $r \in \mathfrak{R}$, r is in the range $[-1, 1]$.

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3. Grey Wolf Optimizer

Grey Wolf Optimizer is based on two pillars: an appropriate hierarchical division of the pack and hunting strategy [7]. Individuals belong to one of four groups: alpha (leader), beta (main helpers), delta (other tasks) and omega (weakest individuals). The hunting process itself can be divided into the following stages:

– tracking and approaching

$$D = |C * X_v(t) - X(t)|, \quad (4)$$

$$C = 2 * r_2, \quad (5)$$

– surrounding and harassing

$$X(t+1) = X_v(t) - A * D, \quad (6)$$

$$A = 2a * r_1 - a, \quad (7)$$

– attack

$$X(t+1) = \frac{X_\alpha + X_\beta + X_\delta}{3}. \quad (8)$$

In the formulas (4)–(8) it is assumed that X and X_v are the locations of wolf and prey, X_α , X_β and X_δ define the location vector of the alpha, beta and delta, r_1 , $r_2 \in [0,1]$ are random numbers and t is the current iteration and coefficient a decrease during calculations from 2 to 0. The algorithm must also be bounded in advance by the maximum number of iterations.

4. 3D crane – mathematical model

3D crane has a cart that moves in the XY plane and a winch with a rope that changes the height on the Z axis of the transported load. The rope can oscillate, so the angles of deflection in two planes are also measured – these are the alpha and beta angles. 3D crane is a multi-input, multi-output system [10, 11]. A miniaturized version was prepared by INTECO (fig. 1) and its model was used to conduct the simulation.

3D crane operates in three dimensions [A1]. A separate control is prepared for each of the axes, which is then fed to a specific motor.

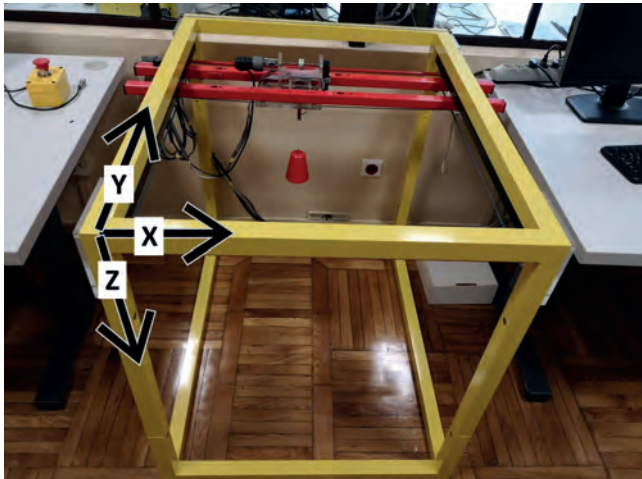


Fig. 1. INTECO 3D crane
Rys. 1. Suwnica 3D firmy INTECO

The following notations were used to describe the system:

x_c – position of the cart on the X axis,

y_c – position of the rail with the carriage on the Y axis,

R – length of the rope on which the payload is suspended,

α – angle between X axis and the rope,

β – angle between the negative direction on the Z axis and the orthogonal projection of the rope on the YZ plane,

x_p – X payload coordinate,

y_c – Y payload coordinate,

z_p – Z payload coordinate,

m_p – payload weight,

m_c – cart weight,

m_r – rail weight,

F_x – the force that drives the cart along the rail,

F_y – the force driving the rail with the trolley,

F_R – the force that drives the rope on which the payload is suspended,

T_x – friction on the X axis,

T_y – friction on the Y axis,

T_R – friction on the Z axis,

k_1 – constant,

k_2 – constant,

k_3 – constant.

It should be assumed that:

$$x_1 = x_c,$$

$$x_2 = \dot{x}_1 = \dot{x}_c,$$

$$x_3 = y_c,$$

$$x_4 = \dot{x}_3 = \dot{y}_c,$$

$$x_5 = \alpha,$$

$$x_6 = \dot{x}_5 = \dot{\alpha},$$

$$x_7 = \beta,$$

$$x_8 = \dot{x}_7 = \dot{\beta},$$

$$x_9 = R,$$

$$x_{10} = \dot{x}_9 = \dot{R}. \quad (9)$$

Then 3D crane can be described by a nonlinear system of equations:

$$\begin{aligned}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= N_1 + \mu_1 c_5 N_3, \\
\dot{x}_3 &= x_4, \\
\dot{x}_4 &= N_2 + \mu_2 s_5 s_7 N_3, \\
\dot{x}_5 &= x_6, \\
\dot{x}_6 &= (s_5 N_1 - c_5 s_7 N_2 + (\mu_1 - \mu_2 s_7^2) c_5 s_5 N_3 + V_5) / x_9, \\
\dot{x}_7 &= x_8, \\
\dot{x}_8 &= -(c_7 N_2 + \mu_2 s_5 c_7 s_7 N_3 + V_6) / (s_5 x_9), \\
\dot{x}_9 &= x_{10}, \\
\dot{x}_{10} &= -c_5 N_1 - s_5 s_7 N_2 - (1 + \mu_1 c_5^2 + \mu_2 s_5^2 s_7^2) N_3 + V_7,
\end{aligned} \tag{10}$$

where:

$$\begin{aligned}
s_n &\equiv \sin x_n, \\
c_n &\equiv \cos x_n, \\
V_5 &= c_5 s_5 x_8^2 x_9 - 2x_{10} x_6 + g c_5 c_7, \\
V_6 &= 2x_8 (c_5 s_6 x_9 + s_5 x_{10}) + g s_7, \\
V_7 &= s_8^2 x_8^2 x_9 + g s_5 c_7 + x_6^2 x_9, \\
\mu_1 &= \frac{m_p}{m_c}, \\
\mu_2 &= \frac{m_p}{m_c + m_r}, \\
N_1 &= u_1 - k_1 x_2, \\
N_2 &= u_2 - k_2 x_4, \\
N_3 &= u_3 - k_3 x_{10}.
\end{aligned} \tag{11}$$

5. Implementation

Figure 2 shows the implementation of the control for the object. The 3D crane model is illustrated by the orange block. There are three inputs:

- control of the motor moving the rail with the cart,
- control of the motor for the cart (moving along the rail),
- control of the motor changing the length of the rope.

Five pieces of information are received at the output:

- position on the X axis,
- position on the Y axis,
- position on the Z axis,
- angle alpha,
- angle beta.

It should be noted that for input X , the values determined by the FOPID 1 and FOPID 4 controllers (parallel arrangement) are summed up. The same procedure was used for input Y . This is due to the system having three inputs and five outputs. Therefore, some information must be considered jointly when generating control. The best approach is to use a combined feedback loop of the X axis with its corresponding α angle and the Y axis with its associated β angle. There is also an option to apply a cascade arrangement, but in this article a parallel setup was used.

The controllers were arranged so that each feedback loop contained only one control type (either PID or FOPID). The focus was on the X and Y axes. The following control scenarios were prepared:

- Two PID controllers before X control, the remaining ones being FOPID,
- Two PID controllers before Y control, the remaining ones being FOPID,
- Five FOPID controllers.

To determine the values of the controllers coefficients, the Grey Wolf Optimizer was used with the adopted cost function (12). This implementation immediately achieved the desired results, so no other methods were chosen. In the future, however, it would be worthwhile to make a comparison with the cost function with squared errors.

$$J = \int_0^{\infty} (|e_x(t)| + |e_y(t)| + |e_z(t)| + |\alpha(t)| + |\beta(t)|) dt, \tag{12}$$

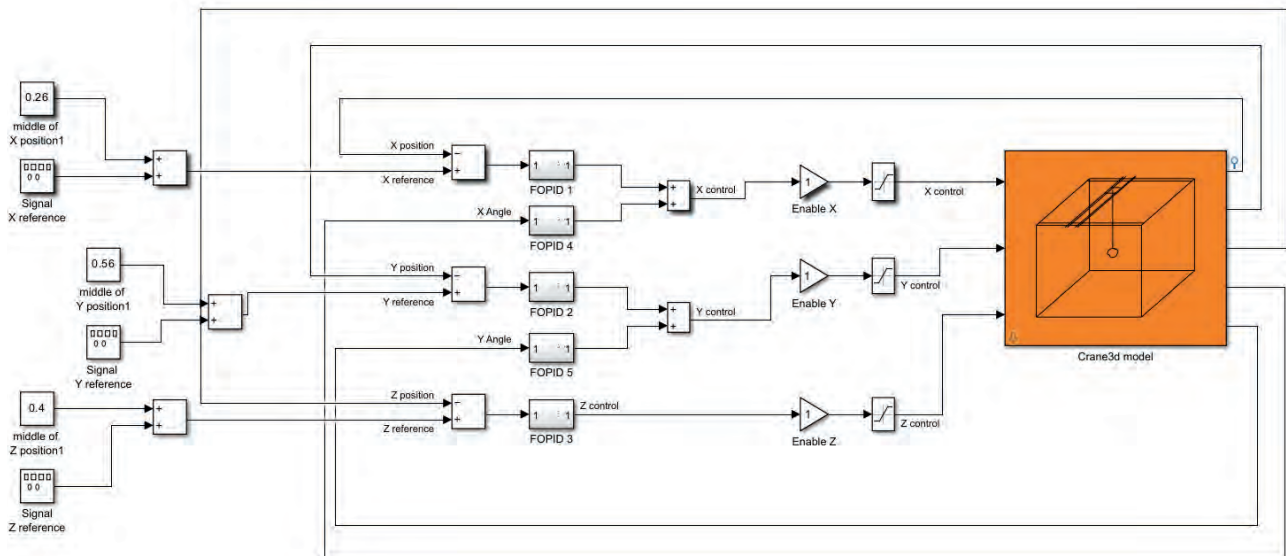


Fig. 2. Simulation model of the 3D crane with FOPID controllers
Rys. 2. Model symulacyjny suwnicy 3D z regulatorami FOPID

where:
 $e_x(t)$ – difference between the set value and the current value on the X axis,
 $e_y(t)$ – difference between the set value and the current value on the Y axis,
 $e_z(t)$ – difference between the set value and the current value on the Z axis,
 $\alpha(t)$ – angle between the Z axis and the X axis,
 $\beta(t)$ – angle between the Z axis and the Y axis.

The algorithm used five wolves and the maximum number of iterations was set to 30. The initial range of coefficients is shown in (13).

$$\begin{aligned} K_P &= [0 : 50] \\ K_I &= [0 : 50] \\ K_D &= [0 : 50] \\ \lambda &= [0 : 1] \\ \mu &= [0 : 1] \end{aligned} \tag{13}$$

6. Results

More information about research can be found in the author’s doctoral dissertation [14].
In the applied configurations, an emphasis was placed on ensuring that the sums of signals were prepared by controllers of one type. For the Z axis, the control value was generated only by FOPID, as controlling this axis is relatively simple. Therefore, three configurations were prepared. Table 1 presents the regulators coefficients determined using the GWO algorithm, which took into account the (12) cost function. The Oustaloup Recursive Approximation was used for the FOPID controllers. Step time 0.0005 seconds was used.
The graph 3 shows position measurements on the X axis. In the first (yellow) and second (green) variants, repeatability in responses to sudden changes in the setpoint (blue) can be observed. In the second control option, double overshoot occurs every time, while in the first version, this phenomenon occurs only once. This causes both systems to approach the reference from

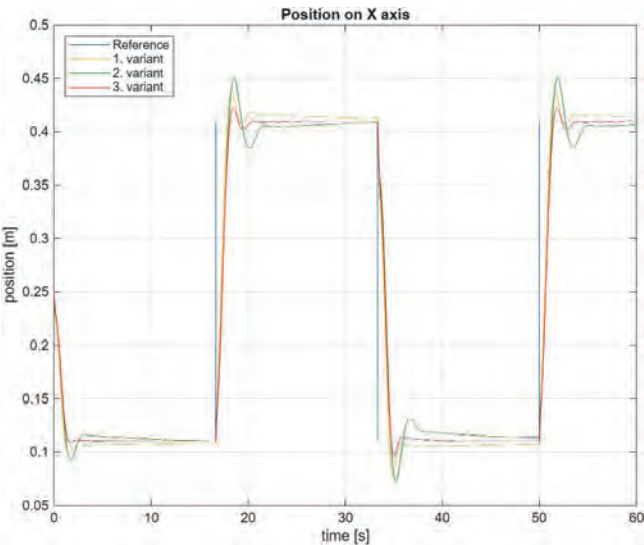


Fig. 3. Simulation output – X position
Rys. 3. Odpowiedź symulacyjna - pozycja X

Table 1. 3D Crane controllers coefficients
Tabela 1. Współczynniki regulatorów dla suwnicy 3D

1. variant	P	I	λ	D	μ
PID 1	131.52	6.58	—	64.49	—
FOPID 2	49.96	5.71	0.43	36.50	0.99
FOPID 3	154.38	1.55	0.051	8.98	0.83
PID 4	146.92	17.58	—	1.12	—
FOPID 5	41.27	35.11	0.50	43.48	0.045
2. variant	P	I	λ	D	μ
FOPID 1	17.55	2.15	0.19	24.83	0.73
PID 2	36.00	1.49	—	18.74	—
FOPID 3	131.61	5.77	0.12	2.04	0.21
FOPID 4	28.28	40.33	0.12	33.64	0.18
PID 5	34.27	5.42	—	1.39	—
3. variant	P	I	λ	D	μ
FOPID 1	48.74	17.96	0.99	12.41	0.99
FOPID 2	7.74	25.64	0.030	18.04	0.99
FOPID 3	116.40	5.75	0.074	32.66	0.058
FOPID 4	29.62	21.28	0.17	20.90	0.14
FOPID 5	28.95	17.23	0.041	8.18	0.31

different directions. Furthermore, the moment of reaching the set point is significantly extended in time. In the third variant, no serious overshoot was observed during the first stabilization. Only in later cases does it appear, but it is substantially smaller. Furthermore, reaching the setpoint is very fast, always taking only a few seconds.
The next figure 4 shows the system’s response on the Y -axis. In the second variant, an overshoot can be observed, after which there is no further approach to the setpoint. A certain steady-state error occurs. For the first and third control options, the behavior is relatively similar. There is a sudden deceleration before the setpoint (slightly earlier in the first variant) and then a gradual, slow approach to the reference. Before the 17th second, the setpoint changed. In this case, the second control option performed best. There was a sharp deceleration before the new setpoint. After the momentary oscillations, stabilization occurred. In the first and third variants, larger overshoots are noticeable. In the first control option, this phenomenon is greater. However, stabilization finally occurs shortly after the 20th second. Further in the graph, the reference changes again. This time, overshoot can be observed in all variants. The remaining behavior is similar to that at the beginning of the graph.
The figure 5 shows the measurement on the Z axis. At the initial reference value, the behaviors of all variants are similar. A rapid approach occurs followed by a sudden deceleration. Steady-state errors can be noticed. In the case of the second variant, after some time, a very slow approach to the setpoint is noticeable. After changing the reference, the approach to the blue graph is very similar for all control options. In the first and second control options, the deceleration is slightly less abrupt and more rounded. For all variants, stabilization occurs on the reference line. However, after some time, a small deviation appears for the second control option.

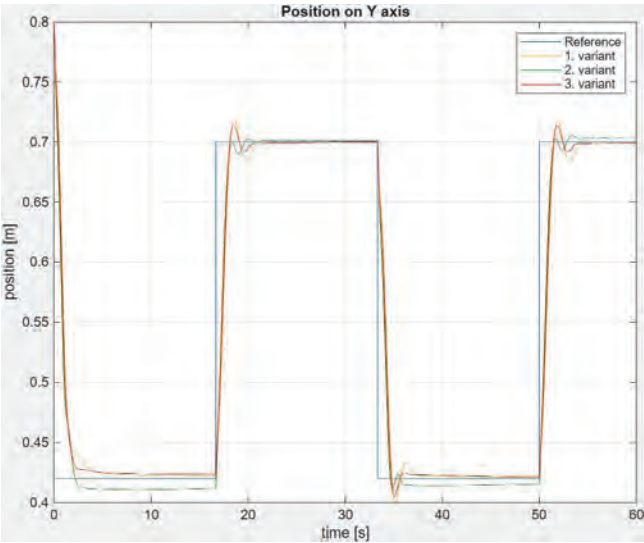


Fig. 4. Simulation output – Y position
Rys. 4. Odpowiedź symulacyjna - pozycja Y

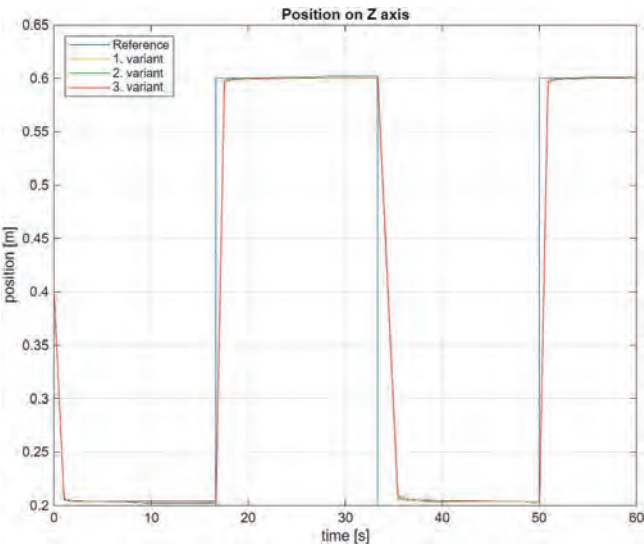


Fig. 5. Simulation output – Z position
Rys. 5. Odpowiedź symulacyjna - pozycja Z

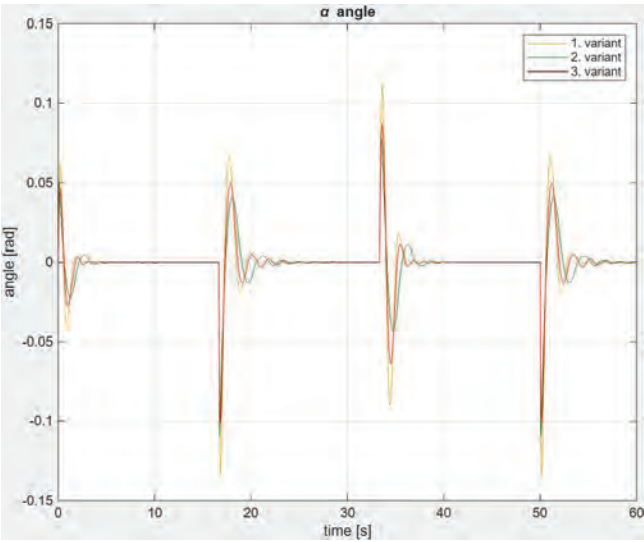


Fig. 6. Simulation output – a position α
Rys. 6. Odpowiedź symulacyjna – kąt α

The graph 6 shows the α angle measurement. The largest deviations were recorded for the first option, with values ranging from approximately -0.12 to approximately 0.11 radians. This approach resulted in faster stabilization around the setpoint, i.e., an angle of 0. The second option had the opposite approach – deviations were usually the smallest, but stabilization took longer. The third control option balanced the two previously mentioned approaches.

The behavior of the β angle is presented in the chart 7. The largest deviations were observed for the second control version, especially at the very beginning of the graph, when values ranged from approximately -0.17 to approximately 0.15 radians. Furthermore, this control variant experienced problems with stabilization around the set angular value after subsequent disturbances. The second and third variants behaved similarly. The minor differences were that the second control variant (relative to the third) was characterized by smaller deviations but a slightly longer stabilization period.

Table 2 contains cost function (12) values calculated for all variants. The third control option achieved the lowest result by far. The high value in the second control version was undoubtedly influenced by problems with fully damping the β angle oscillations. Furthermore, there was a small error on the Y axis. In this option, the PID controllers, which generated control for the Y axis motor, had the greatest impact. In the case of the first control option, the result was certainly related to the X axis position – difficulties in traversing the last few centimeters to the setpoint.

Table 2. Cost function values (12) for a 3D crane
Tabela 2. Wartości funkcji kosztu (12) dla suwnicy 3D

	Cost function value
1. variant	7497.05
2. variant	8066.55
3. variant	7051.12

The controllers' calculation times for the third variant were also measured. The information was summarized in 8-12 graphs and the 3 table. Calculations for FOPID 4 and FOPID 5 controllers were slightly slower. This is likely due to the fact that they use unprocessed angular data, allowing them

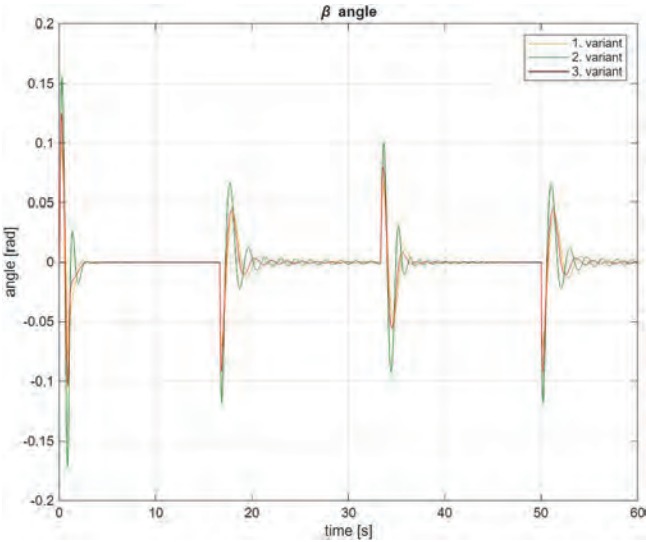


Fig. 7. Simulation output – β position
Rys. 7. Odpowiedź symulacyjna – kąt β

Table 3. Average and maximum calculation time of controllers – third variant

Tabela 3. Średnie i maksymalne wartości czasów wykonania regulatorów - trzeci wariant

Controller	Average [s]	Maximum [s]
FOPID 1	2.5029e-07	5.5610e-06
FOPID 2	2.8422e-07	4.7885e-06
FOPID 3	3.1299e-07	3.4473e-06
FOPID 4	3.5585e-07	3.0823e-06
FOPID 5	4.0239e-07	2.9723e-06

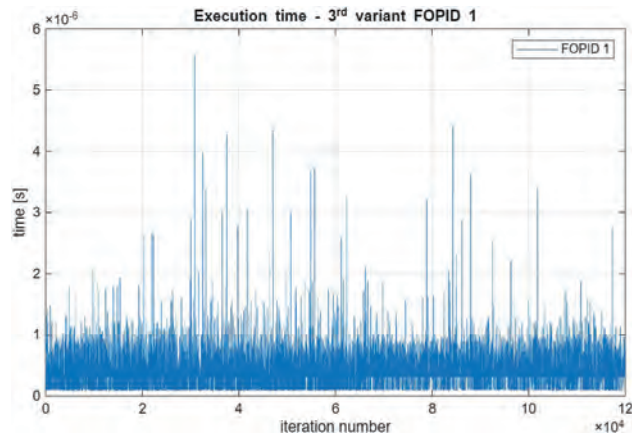


Fig. 8. Execution time - third variant FOPID 1
Rys. 8. Czas wykonania - trzeci wariant FOPID 1

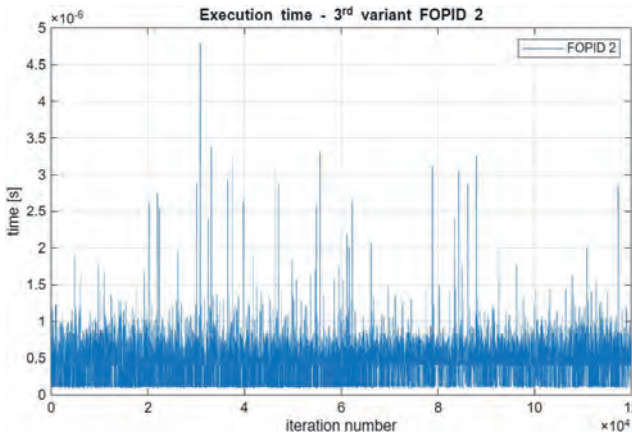


Fig. 9. Execution time - third variant FOPID 2
Rys. 9. Czas wykonania - trzeci wariant FOPID 2

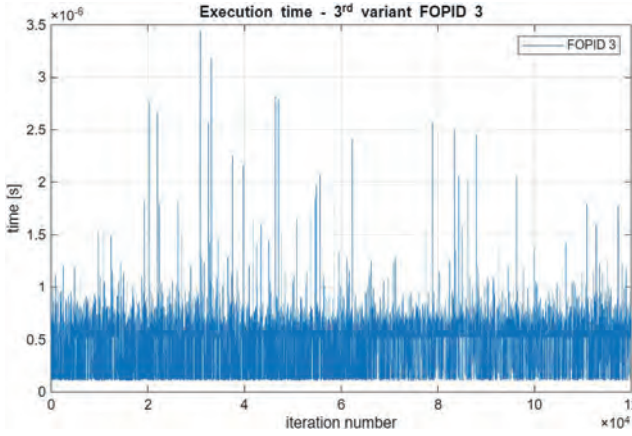


Fig. 10. Execution time - third variant FOPID 3
Rys. 10. Czas wykonania - trzeci wariant FOPID 3

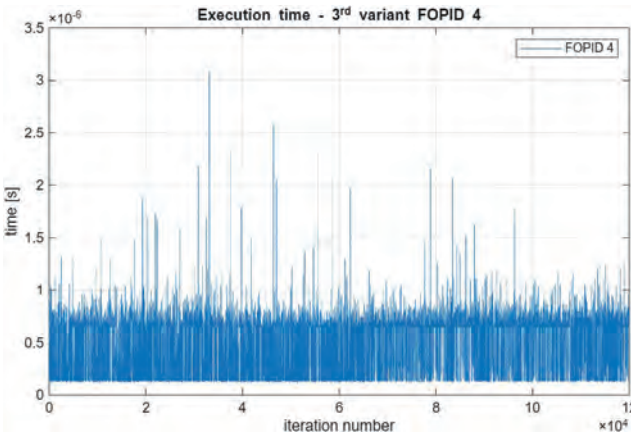


Fig. 11. Execution time - third variant FOPID 4
Rys. 11. Czas wykonania - trzeci wariant FOPID 4

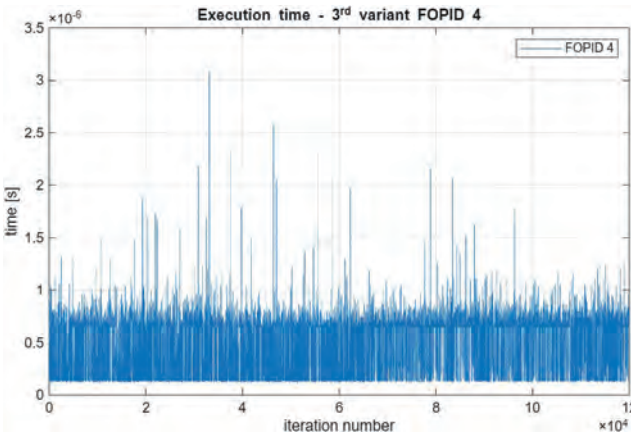


Fig. 12. Execution time - third variant FOPID 5
Rys. 12. Czas wykonania - trzeci wariant FOPID 5

to begin calculations earlier. The remaining controllers must wait a while for error values to be determined. Since FOPID 4 and FOPID 5 started their work faster, they finish computations earlier. This means that the processor’s computing power begins to be released and can be used by FOPID 1, FOPID 2 and FOPID 3. Moreover, the computer probably optimizes the calculations so that all information enters the object at approximately the same time.

7. Conclusions

This paper presents three control variants for a 3D crane system, differing in the number of FOPID and PID controllers and their arrangement. The coefficients were optimized using the Grey Wolf Optimizer.

The determined cost function values (12) indicate that, in the given scenario, control version with only FOPID controllers yielded the best results. Other variants experienced some issues, such as problems with damping the beta angle oscillations in the second variant.

Furthermore, the controllers’ computation times were measured for the third control option — five FOPID controllers. Based on the results, even the maximum values were well below the time step of 0.0005. This indicates that these types of controllers can be used more broadly in systems where real-time control is essential.

In the future, the author plans to add a check to the GWO algorithm at each loop iteration to ensure that the K_p , K_I and

K_D values fall within a specified range. This will force the algorithm to search for smaller coefficients. Another direction of development could be the use of other cost functions, for example, squared error functions. Additionally, it would be worthwhile to add an additional scenario in future research for comparative purposes, in which only PID controllers were used.

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Klasyfikacja regulatorów FOPID i PID ze względu na typ sygnału wejściowego na podstawie badań symulacyjnych suwnicy 3D

Streszczenie: Suwnica 3D jest systemem służącym do przemieszczania ciężkich ładunków w określonym obszarze roboczym. Do jej sterowania zastosowano pięć regulatorów. Trzy z nich odpowiadają za regulację położenia wzdłuż osi X, Y i Z, natomiast pozostałe dwa wykorzystują informacje o odchyleniach wyrażonych wartościami kątów alfa i beta. Celem artykułu było przeprowadzenie badań symulacyjnych mających na celu weryfikację możliwości sterowania suwnicą 3D za pomocą regulatorów FOPID. Opracowano trzy warianty układu sterowania, różniące się liczbą oraz rozmieszczeniem regulatorów PID i FOPID. Ponadto przeanalizowano możliwość zastosowania tego rodzaju regulacji w systemach czasu rzeczywistego. W związku z tym przeprowadzono pomiary czasu obliczeń dla wariantu wykorzystującego wyłącznie regulatory FOPID.

Słowa kluczowe: suwnica 3D, FOPID, PID, czas wykonania, wymagania czasu rzeczywistego

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