Simple Stability Conditions of Linear Switched Systems with Time-Dependent Switching

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Abstract: In this paper, the stability of continuous-time and discrete-time linear switched systems with time-dependent switching is investigated. Solutions to the state equations are provided and sufficient conditions for the stability of such systems are established. The stability conditions presented for continuous-time systems are valid in the following cases: 1) there are no specific restrictions on the switching function if each switching interval is equal to or greater than 1 second; 2) the switching function has to be a periodic function (or a function describing permutations of identical sequences of subsystems) if at least one of the switching intervals is less than 1 second. In the case of discrete-time systems the switching function is arbitrary. The effectiveness of the presented approach is demonstrated by numerical examples.

Keywords: continuous-time, discrete-time, linear switched system, stability, time-dependent switching

1. Introduction

The dynamics of many real physical phenomena can be described by a set of subsystems and a logical rule that controls the switching between them [12]. Mathematical models of such systems are called switched systems. This class of dynamical systems has been considered since the end of the 20th century [4, 13].

The theory of switched systems is widely used in many engineering problems related to mechanical systems, power electronic converters, electric power systems, multi-agent systems, the automotive industry, aircraft and air traffic control, and many other fields, see, e.g., [1, 5, 8, 16, 20].

Stability is one of the most important concepts in dynamical systems theory. The stability of the considered class of systems has been studied in many papers and books for: standard switched systems [4, 12–14, 17, 19, 22], positive switched systems [7, 25], singular switched systems [2, 26], as well as hybrid systems [4, 15, 17]. Moreover, in recent years, fractional-order switched systems have attracted much attention from researchers and problems concerning the stability of such systems have also been studied [6, 9, 21, 23].

The literature also includes many works devoted to the control and stabilization of switched systems [12, 18, 24].

In this paper, the stability of continuous-time and discrete-time linear switched systems with time-dependent switching will be investigated and some simple stability conditions for this class of dynamical systems will be established. It is assumed that it is possible to determine the relationship between the total switch-on time of individual subsystems. Moreover, the conditions presented for continuous-time systems are valid in the following cases: 1) there are no specific restrictions on the switching function if each switching interval is equal to or greater than 1 second; 2) the switching function has to be a periodic function (or a function describing permutations of identical sequences of subsystems) if at least one of the switching intervals is less than 1 second. In the case of discrete-time systems the switching function is arbitrary.

The paper is organized as follows. In Section 2 considered state-space models of switched linear systems are introduced and solutions to the state equations of such systems are provided. Section 3 is devoted to the stability analysis of switched systems. In this section, sufficient conditions for the stability are established. Numerical examples are presented in Section 4. Concluding remarks are given in Section 5.

The following notation will be used: $\mathbb{R}$ – the set of real numbers, $\mathbb{R}^{n\times m}$ – the set of $n \times m$ real matrices and $\mathbb{R}^n = \mathbb{R}^{n\times 1}$, $\mathbb{Z}_{\geq 0}$ – the set of nonnegative integers.

2. Preliminaries

In this section considered state-space models will be presented and solutions to the state equations will be provided.

2.1. Continuous-time linear switched systems

Consider the continuous-time linear switched state-space model in the form

$$\dot{x}(t) = A_{\sigma(t)}x(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A_{\sigma(t)} \in \mathbb{R}^{n\times n}$, $\sigma(t)$ is the piecewise constant switching function which takes values in

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the finite set $\{1, 2, \ldots, N\}$ and N is the number of subsystems. It is assumed that there are no jumps of the state variables at the switching instants t_i, $i \in \mathbb{Z}_{\geq 0}$ and the $\sigma(t_i)$-th system is active in the interval $t \in [t_i, t_{i+1})$.

It is well-known [10] that the solution to the equation (1) in the interval $t \in [t_i, t_{i+1})$ is given by

$$x(t) = e^{A_{\sigma(t_i)}(t-t_i)} x_i, \quad (2)$$

where $x_i = x(t_i) \in \mathbb{R}^n$ is the initial condition at the switching instant t_i, $i \in \mathbb{Z}_{\geq 0}$.

From (2) it follows that the initial conditions at the switching instants t_i, $i \in \mathbb{Z}_{\geq 0}$ can be computed using the following recursive formula

$$x_{i+1} = x(t_{i+1}) = e^{A_{\sigma(t_i)}\Delta t_i} x_i, \quad (3)$$

where

$$\Delta t_i = t_{i+1} - t_i. \quad (4)$$

Using the equation (3) for $i = 0$ we obtain

$$x_1 = e^{A_{\sigma(t_0)}\Delta t_0} x_0. \quad (5)$$

From the formula (3) for $i = 1$ and (5) we have

$$x_2 = e^{A_{\sigma(t_1)}\Delta t_1} x_1 = e^{A_{\sigma(t_1)}\Delta t_1} e^{A_{\sigma(t_0)}\Delta t_0} x_0. \quad (6)$$

We can continue this procedure for $i \geq 2$.

Therefore, the initial condition $x_i \in \mathbb{R}^n$ at i-th switching instant t_i, $i \in \mathbb{Z}_{\geq 0}$ for known initial condition $x_0 \in \mathbb{R}^n$ and previous switching instants $t_0, \ldots, t_{i-1}$ is given by the formula

$$x_i = e^{A_{\sigma(t_{i-1})}\Delta t_{i-1}} e^{A_{\sigma(t_{i-2})}\Delta t_{i-2}} \ldots e^{A_{\sigma(t_0)}\Delta t_0} x_0, \quad (7)$$

where Δt_i is defined by (4).

Thus, from equations (2) and (7) we obtain the following solution to the state equation (1).

Lemma 1. The solution to the equation (1) for known initial condition $x_0 \in \mathbb{R}^n$ and switching instants t_i, $i \in \mathbb{Z}_{\geq 0}$ is given by the formula

$$x(t) = e^{A_{\sigma(t_i)}(t-t_i)} e^{A_{\sigma(t_{i-1})}(\Delta t_{i-1})} \ldots e^{A_{\sigma(t_0)}(\Delta t_0)} x_0, \quad (8)$$

where Δt_i is defined by (4).

2.2. Discrete-time linear switched systems

Consider the discrete-time linear switched state-space model in the form

$$\bar{x}_{k+1} = \bar{A}_{\sigma_k} \bar{x}_k, \qquad k \in \mathbb{Z}_{\geq 0}, \quad (9)$$

where $\bar{x}_k \in \mathbb{R}^n$ is the state vector, $\bar{A}_{\sigma_k} \in \mathbb{R}^{n \times n}$, σ_k is the switching function which takes values in the finite set $\{1, 2, \ldots, N\}$ and N is the number of subsystems. It is assumed that the σ_{k_i}-th system is active in the interval $k \in [k_i, k_{i+1})$ and the initial condition of that system is the final value of the state vector in the previous switching interval $k \in [k_{i-1}, k_i)$.

It is well-known [10] that the solution to the equation (9) in the interval $k \in [k_i, k_{i+1})$ is given by

$$\bar{x}_k = \bar{A}_{\sigma_{k_i}}^{k-k_i} \bar{x}_i, \quad (10)$$

where $\bar{x}_i = \bar{x}(k_i) \in \mathbb{R}^n$ is the initial condition at the switching instant k_i, $i \in \mathbb{Z}_{\geq 0}$.

For discrete-time systems, one can present considerations analogous to those introduced in Section 2.1. Therefore, the initial conditions $\bar{x}_i \in \mathbb{R}^n$ at the switching instants k_i, $i \in \mathbb{Z}_{\geq 0}$ can be computed either from the recursive formula

$$\bar{x}_{i+1} = \bar{x}(k_{i+1}) = \bar{A}_{\sigma_{k_i}}^{\Delta k_i} \bar{x}_i \quad (11)$$

or the equation

$$\bar{x}_i = \bar{A}_{\sigma_{k_{i-1}}}^{\Delta k_{i-1}} \bar{A}_{\sigma_{k_{i-2}}}^{\Delta k_{i-2}} \ldots \bar{A}_{\sigma_{k_0}}^{\Delta k_0} \bar{x}_0, \quad (12)$$

where

$$\Delta k_i = k_{i+1} - k_i. \quad (13)$$

Thus, from (10) and (12) we obtain the following solution to the state equation (9).

Lemma 2. The solution to the equation (9) for known initial condition $\bar{x}_0 \in \mathbb{R}^n$ and switching instants k_i, $i \in \mathbb{Z}_{\geq 0}$ is given by the formula

$$\bar{x}_k = \bar{A}_{\sigma_{k_i}}^{k-k_i} \bar{A}_{\sigma_{k_{i-1}}}^{\Delta k_{i-1}} \bar{A}_{\sigma_{k_{i-2}}}^{\Delta k_{i-2}} \ldots \bar{A}_{\sigma_{k_0}}^{\Delta k_0} \bar{x}_0, \quad (14)$$

where Δk_i is defined by (13).

3. Stability of linear switched systems

In this section sufficient conditions for stability of continuous-time and discrete-time switched linear systems will be established.

To obtain the stability conditions the following norms will be used:

1) ∞-norm of a vector $x = [x_i] \in \mathbb{R}^n$

$$\|x\| = \max_{1 \leq i \leq n} |x_i|, \quad (15)$$

2) ∞-norm of a matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$

$$\|A\| = \max_{1 \leq i \leq n} \left(\sum_{j=1}^{n} |a_{ij}| \right). \quad (16)$$

In the following considerations we will use well-known operations on norms of vectors and matrices [11].

We also assume that it is possible to determine the relationship between the total switch-on time of individual subsystems of the switched systems (1) and (9). Moreover, the conditions presented for continuous-time systems are valid in the following cases:

1) there are no specific restrictions on the switching function $\sigma(t)$ if each switching interval $\Delta t_i \geq 1$, $i \in \mathbb{Z}_{\geq 0}$;
2) the switching function has to be a periodic function (or a function describing permutations of identical sequences of subsystems) if at least one of the switching intervals $\Delta t_i < 1$.

In the case of discrete-time systems the switching function is arbitrary.

If the number of switching instants is finite, testing the stability of the switched system is reduced to testing the stability of the last active subsystem using methods known in the literature for standard linear systems [3, 10].

3.1. Stability of continuous-time switched systems

Definition 1. The continuous-time switched system (1) is called asymptotically stable if

$$\lim_{t\to\infty} x(t) = 0, \tag{17}$$

for all initial conditions $x_0 \in \mathbb{R}^n$.

From (8) we have

$$\begin{aligned}\lim_{t\to\infty}\left\|x(t)\right\| &= \lim_{t\to\infty}\left\|e^{A_{\sigma(t_i)}(t-t_i)}\dots e^{A_{\sigma(t_0)}(\Delta t_0)}x_0\right\| \\ &\le \prod_{i=0}^{\infty}\left(\left\|e^{A_{\sigma(t_i)}\Delta t_i}\right\|\right)\left\|x_0\right\|.\end{aligned} \tag{18}$$

Let us introduce $T_j \ge 0$, $T_j > 0$, $j = 1, \ldots, N$, which denotes the total switch-on time of the j-th subsystem and observe that

$$t = \sum_{j=1}^{N} T_j = \sum_{j=1}^{N} c_j t, \qquad \sum_{j=1}^{N} c_j = 1, \tag{19}$$

where $c_j \ge 0$, $j = 1, \ldots, N$ is the relative total switch-on time of the j-th subsystem. Therefore, the inequality (18) can be rewritten as

$$\begin{aligned}\lim_{t\to\infty}\left\|x(t)\right\| &\le \prod_{j=1}^{N}\left(\left\|e^{A_j}\right\|^{T_j}\right)\left\|x_0\right\| \\ &= \prod_{j=1}^{N}\left(\left\|e^{A_j}\right\|^{c_j t}\right)\left\|x_0\right\|.\end{aligned} \tag{20}$$

The above considerations are valid in the following two cases:

1) for the switching intervals satisfying $\Delta t_i \ge 1$, $i \in \mathbb{Z}_{\ge 0}$ we have

$$\left\|e^{A_{\sigma(t_i)}\Delta t_i}\right\| \le \left\|e^{A_{\sigma(t_i)}}\right\|^{\Delta t_i} \tag{21}$$

and the inequality (20) holds for an arbitrary switching function $\sigma(t)$;

2) for any switching interval satisfying $\Delta t_i < 1$ we have

$$\left\|e^{A_{\sigma(t_i)}\Delta t_i}\right\| \ge \left\|e^{A_{\sigma(t_i)}}\right\|^{\Delta t_i} \tag{22}$$

and the inequality (20) holds only for a periodic switching function $\sigma(t)$ (or a switching function describing permutations of identical sequences of subsystems).

Therefore, we obtain $\lim_{t\to\infty}\left\|x(t)\right\| = 0$ if

$$\lim_{t\to\infty}\prod_{j=1}^{N}\left\|e^{A_j}\right\|^{c_j t} = 0 \tag{23}$$

and this implies

$$\prod_{j=1}^{N}\left\|e^{A_j}\right\|^{c_j} < 1. \tag{24}$$

Thus, the following theorem has been proved.

Theorem 1. The continuous-time switched system (1) is asymptotically stable if the switching function $\sigma(t)$:

1) is any function satisfying the condition (24) for $\Delta t_i \ge 1$, $i \in \mathbb{Z}_{\ge 0}$;
2) is a periodic function (or a function describing permutations of identical sequences of subsystems) satisfying the condition (24) for at least one $\Delta t_i < 1$, $i \in \mathbb{Z}_{\ge 0}$.

3.2. Stability of discrete-time switched systems

Definition 2. The discrete-time switched system (9) is called asymptotically stable if

$$\lim_{k\to\infty} \overline{x}_k = 0 \tag{25}$$

for all initial conditions $\overline{x}_0 \in \mathbb{R}^n$.

The analysis can be performed in a similar way as for continuous-time systems. From (14) we have

$$\begin{aligned}\lim_{k\to\infty}\left\|\overline{x}_k\right\| &= \lim_{k\to\infty}\left\|\overline{A}_{\sigma_{k_i}}^{k-k_i}\overline{A}_{\sigma_{k_{i-1}}}^{\Delta k_{i-1}}\dots\overline{A}_{\sigma_{k_0}}^{\Delta k_0}\right\| \\ &\le \prod_{i=0}^{\infty}\left(\left\|\overline{A}_{\sigma_{k_i}}^{\Delta k_i}\right\|\right)\left\|\overline{x}_0\right\|.\end{aligned} \tag{26}$$

Introducing $K_j \ge 0$, $j = 1, \ldots, N$ as the total switch-on time of the j-th subsystem and noticing that

$$k = \sum_{j=1}^{N} K_j = \sum_{j=1}^{N} d_j k, \qquad \sum_{j=1}^{N} d_j = 1, \tag{27}$$

where $d_j \ge 0$, $j = 1, \ldots, N$ is the relative total switch-on time of the j-th subsystem, the inequality (26) can be rewritten as

$$\begin{aligned}\lim_{k\to\infty}\left\|\overline{x}_k\right\| &\le \prod_{j=1}^{N}\left(\left\|\overline{A}_j\right\|^{K_j}\right)\left\|\overline{x}_0\right\| \\ &= \prod_{j=1}^{N}\left(\left\|\overline{A}_j\right\|^{d_j k}\right)\left\|\overline{x}_0\right\|.\end{aligned} \tag{28}$$

The switching intervals always satisfy the condition $\Delta k_i \ge 1$, $i \in \mathbb{Z}_{\ge 0}$ since $k \in \mathbb{Z}_{\ge 0}$. Therefore, the inequality (28) always holds for an arbitrary switching function σ_k since

$$\left\|\overline{A}_{\sigma_{k_i}}^{\Delta k_i}\right\| \le \left\|\overline{A}_{\sigma_{k_i}}\right\|^{\Delta k_i} \tag{29}$$

for $\Delta k_i \ge 1$, $i \in \mathbb{Z}_{\ge 0}$.

Thus, we obtain $\lim_{k\to\infty}\left\|\overline{x}_k\right\| = 0$ if

$$\lim_{k\to\infty}\prod_{j=1}^{N}\left\|\overline{A}_j\right\|^{d_j k} = 0 \tag{30}$$

and this implies

$$\prod_{j=1}^{N}\left\|\overline{A}_j\right\|^{d_j} < 1. \tag{31}$$

Therefore, the following theorem has been proved.

Theorem 2. The discrete-time switched system (9) is asymptotically stable if the switching function σ_k is any function satisfying the condition (31).

3.3. Discussion

The conditions given in Theorems 1 and 2 take advantage of the fact that the stability of switched systems depends mainly on the form of the switching function. The topic of stable switched systems composed of stable and unstable subsystems is well-known and widely discussed in the literature [14]. Therefore, an adequate ratio of the switch-on times of the individual subsystems must be ensured.

Due to the above, methods based on analyzing the position of the system poles on the complex plane do not apply to switched systems. Thus, the conditions proposed in the article are based on bounding the norm of the state vector. The undoubted advantage of this approach is its simplicity allowing the investigation of switched systems with various configurations, consisting of both stable and unstable subsystems, as long as the condition of the switch-on times ratio for individual subsystems is met.

However, the actual stability region boundaries of the switched system may be located further than the calculations result, as the conditions are obtained through the use of matrix norms inequalities. Therefore, it is an open problem to provide the necessary conditions for the stability of switched systems using the presented method. Furthermore, extending the analysis to a wider group of switching functions for the case of continuous-time systems with faster switching (with time intervals less than 1 second) is also an open issue.

4. Numerical examples

Example 1. Consider the continuous-time linear switched system (1) consisting of $N = 3$ subsystems with the state matrices given by

$$A_1 = \begin{bmatrix} -3 & 0 & 2 \\ 3 & -4 & 6 \\ -2 & -1 & -5 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 3 & -2 & 0 \\ 2 & -3 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -5 & 1 & 0 \\ 3 & -2 & 1 \\ 1 & 0 & 2 \end{bmatrix}. \tag{32}$$

It is easy to check that only the first subsystem is stable, while the other two are unstable. We compute the norms

$$\left\| e^{A_1} \right\| = 0.045, \quad \left\| e^{A_2} \right\| = 18.1813, \quad \left\| e^{A_3} \right\| = 9.0262. \tag{33}$$

From (19), (24) and (33) we obtain

$$\begin{cases} 0.045^{c_1} \cdot 18.1813^{c_2} \cdot 9.0262^{c_3} < 1 \\ c_1 + c_2 + c_3 = 1. \end{cases} \tag{34}$$

Therefore, the switched system (1) with (32) will be asymptotically stable if the switching function $\sigma(t)$ satisfies the condition (34), i.e., the ratio of the switch-on times of subsystems will be $c_1 : c_2 : c_3$ such that (34) holds.

For further analysis we assume $c_1 = 0.5$, $c_2 = 0.2$, $c_3 = 0.3$ satisfying (34) and we define the switching function as

$$\sigma(t) = \begin{cases} 2, & t \in \left[l, l+0.2\right), \\ 1, & t \in \left[l+0.2, l+0.7\right), \\ 3, & t \in \left[l+0.7, l+1\right), \end{cases} \quad l \in \mathbb{Z}_{\geq 0}. \tag{35}$$

In Figure 1 we present time plots of the state variables of the continuous-time linear switched system (1) with (32) for $x_0 = [\,4\ 2\ 5\,]^{\mathrm{T}}$ and the time plot of the switching function $\sigma(t)$ given by (35). We can see that the state variables tend to zero. Similar results can be obtained for any other initial condition x_0 and for any switching function $\sigma(t)$ satisfying the condition (34).

Example 2. Consider the discrete-time linear switched system (9) consisting of $N = 2$ subsystems with the state matrices given by

$$\bar{A}_1 = \begin{bmatrix} 0.7 & 1.1 & 0.4 \\ 0.8 & 0.9 & 0.2 \\ 0.5 & 1.2 & 0.1 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} 0.6 & 0 & 0.1 \\ 0.3 & 0.3 & 0 \\ 0.1 & 0.1 & 0.4 \end{bmatrix}. \tag{36}$$

It is easy to check that the first subsystem is unstable and the second one is stable. We compute the norms

$$\left\| \bar{A}_1 \right\| = 2.2, \quad \left\| \bar{A}_2 \right\| = 0.7. \tag{37}$$

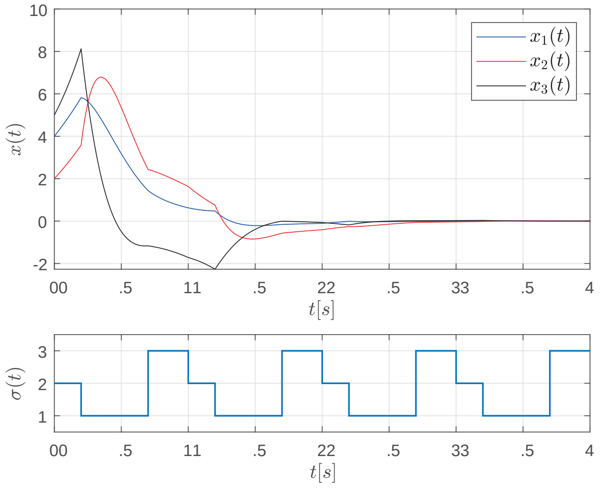

Fig. 1. State variables (up) and the switching function (down) of the continuous-time linear switched system (1) with (32) for $x_0 = [\,4\ 2\ 5\,]^{\mathrm{T}}$
Rys. 1. Zmienne stanu (u góry) i funkcja przełączająca (u dołu) liniowego ciągłego układu przełączalnego (1) o macierzach (32) dla $x_0 = [\,4\ 2\ 5\,]^{\mathrm{T}}$

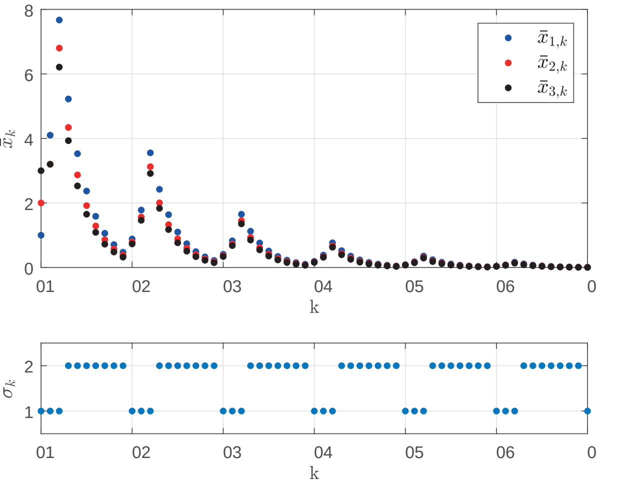

Fig. 2. State variables (up) and the switching function (down) of the discrete-time linear switched system (9) with (36) for $\bar{x}_0 = [\,1\ 2\ 3\,]^{\mathrm{T}}$
Rys. 2. Zmienne stanu (u góry) i funkcja przełączająca (u dołu) liniowego dyskretnego układu przełączalnego (9) o macierzach (36) dla $\bar{x}_0 = [\,1\ 2\ 3\,]^{\mathrm{T}}$

From (27), (31) and (37) we obtain

$$\begin{cases} 2.2^{d_1} \cdot 0.7^{d_2} < 1 \\ d_1 + d_2 = 1. \end{cases} \tag{38}$$

Therefore, the switched system (9) with (36) will be asymptotically stable if the switching function σ_k satisfies the condition (38), i.e., the ratio of the switch-on times of subsystems will be $d_1 : d_2$ such that (38) holds.

For further analysis we assume $d_1 = 0.3$, $d_2 = 0.7$ satisfying (38) and we define the switching function as

$$\sigma_k = \begin{cases} 1, & k \bmod 10 \in \{0,1,2\}, \\ 3, & k \bmod 10 \in \{3,4,\ldots,9\}, \end{cases} \tag{39}$$

where mod denotes the modulo operation.

In Figure 2 we present time plots of the state variables of the discrete-time linear switched system (9) with (36) for $\overline{x}_0 = [\ 1\ 2\ 3\ ]^{\mathrm{T}}$ and the time plot of the switching function σ_k given by (39). We can see that the state variables tend to zero. Similar results can be obtained for any other initial condition $\overline{x}_0$ and for any switching function σ_k satisfying the condition (38).

5. Concluding remarks

In this paper, the stability of continuous-time and discrete-time linear switched systems with time-dependent switching has been investigated. Solutions to the state equations of such systems have been provided (Lemmas 1 and 2). The main result of the paper is the establishment of sufficient conditions for the stability of the continuous-time (Theorem 1) and discrete-time (Theorem 2) linear switched systems. The effectiveness of the presented approach has been demonstrated by numerical examples.

The conditions proposed in the article are based on the matrix and vector norms calculus since the methods using the analysis of the position of the system poles on the complex plane do not apply to switched systems. The undoubted advantage of this approach is its simplicity allowing the investigation of switched systems with various configurations, consisting of both stable and unstable subsystems, as long as the condition of the switch-on times ratio for individual subsystems is met.

The considerations can be further extended to fractional-order linear switched systems. Establishing the necessary conditions for the stability of this class of dynamical systems based on the presented methodology also remains an open problem. Furthermore, extending the analysis to a wider group of switching functions for the case of continuous-time systems with faster switching (with time intervals less than 1 second) is also an open issue.

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Proste warunki stabilności liniowych układów przełączalnych z przełączaniem zależnym od czasu

Streszczenie: W artykule zbadano stabilność ciągłych i dyskretnych liniowych układów przełączalnych z przełączaniem zależnym od czasu. Podano rozwiązania równań stanu i wyznaczono warunki wystarczające stabilności takich układów. Warunki przedstawione dla układów ciągłych są słuszne w następujących przypadkach: 1) nie ma szczególnych ograniczeń na funkcję przełączania, jeżeli każdy przedział czasu w kolejnych przełączeniach jest nie krótszy niż 1 sekunda; 2) funkcja przełączająca musi być funkcją okresową (lub funkcją opisującą permutacje identycznych sekwencji załączanych podukładów), jeżeli co najmniej jeden z przedziałów czasu w kolejnych przełączeniach jest krótszy od 1 sekundy. W przypadku układów dyskretnych funkcja przełączająca jest dowolna. Skuteczność zaprezentowanego podejścia pokazano na przykładach numerycznych.

Słowa kluczowe: ciągły, dyskretny, liniowy, układ przełączalny, stabilność, przełączanie zależne od czasu

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