Evolutionary method of robust controller computation

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Abstract: Mathematical methods of robust controller coefficients selection in H_∞ spaces are very complicated. A control system integrator has to know functional analysis methods. To solve this kind of problem, evolutionary algorithms can be used. The paper presents both the method and simulation results of evolutionary algorithms application for a robust controller coefficients selection. To select robust controller, only two requirements are used: stability check and geometric dependency – minimizing the maximum distance between Nyquist diagrams of operations – G($j\omega$) and 1/F($j\omega$). Where G($j\omega$) and F($j\omega$) are controller and plant transfer functions in a feedback control system.

Keywords: genetic algorithms, robust control

1. Concept of control system optimality

Let the object be described by a linear operation F transforming a set of signals belonging to Banach space V into themselves. Let's denote as V(F) the set of all points $x \in V$ where $F^{-1}(x) \in V$ and $||F^{-1}(x)|| < \infty$.

Let's consider a system with feedback, as shown in the diagram, i.e. described with these equations:

$$\begin{gathered} y = G(x) \\ x = z - F(y) \end{gathered} \tag{1}$$

where F and G are the operations transforming Banach space V into itself. Whereas z and x are the set signal and the error signal, respectively. Equations (1) can be noted in the form of:

$$z - F(g(x)) = x \tag{2}$$

or alternatively

$$z = x + F(G(x)) \tag{3}$$

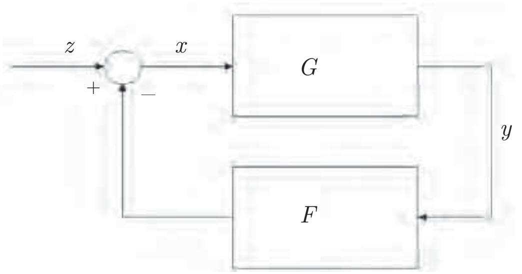

Fig. 1. Control system diagram
Rys. 1. Schemat układu regulacji

If there is a solution $x \in V$ to the eq. (2) for $z \in V(F) \subset V$, then $x \in V(F)$. It results from the fact, that $z \in V(F)$ and $F(G(x)) \in V(F)$. Since each of the elements of the eq. (3) belongs to $V(F)$, performing the operation F^{-1} for both sides, we can write

$$F^{-1}(z) = F^{-1}(x) + G(x) \tag{4}$$

We will determine the operations in a natural way in the set of all functions transforming vector space X to space Y. Therefore, equation (4) can be noted as follows:

$$F^{-1}(z) = (F^{-1} + G)(x) \tag{5}$$

Assuming additionally that $||(F^{-1} + G)^{-1}|| < \infty$, we can note further

$$(F^{-1} + G)^{-1} (F^{-1}(z)) = x \tag{6}$$

Calculating norms for both sides of the equation (6), we obtain

$$||x|| = ||(F^{-1}+G)^{-1}(F^{-1}(z))|| \leq ||(F^{-1} + G)^{-1}|| \, ||(F^{-1}(z))|| \tag{7}$$

which can be further noted as

$$||x|| \leq ||(F^{-1} + G)^{-1}|| \, ||F^{-1}(z)|| \tag{8}$$

The limitation, $||F^{-1}(z)|| < \infty$, resulting from the affinity of the signal to the set $V(F)$ is a natural limitation since an "ideal" control system performs, approximately, an operation reverse to operation F.

The condition, $||x|| \leq ||(F^{-1} + G)^{-1}|| < \infty$, is most often the sufficient condition for the existence of solutions to equations (1) in spaces L^2 or M for $z \in V(F)$.

Following the above considerations, we can formulate the following statement:

Let C denote a set of controllers, G, which can be used in the system in question. The H_∞ optimization theory assumes that $C = RH_\infty$ (the controller is given by the measurable transmittance with real coefficients and limited to $re\ s \geq 0$).

We shall call controller $G^* \in R$ the optimal one for constraints belonging to set $V(F)$ if for each $G_1 \in R$ the following is true:

$$\sup_{z \in V(F)} \left\| x_{G^*} \right\| \leq \sup_{z \in V(F)} \left\| x_{G_1} \right\|$$

Theorem. Let operation F describing the object, and operations $G \in C$ belonging to the set of controllers C transform the set of signal from Banach space R ($L^2(0,\infty)$ or M) into themselves. If, for controller $G^* \in C$, the expression:

$$r(G) = \inf_{res \geq 0} \left| \frac{1}{F(s)} + G(s) \right| \tag{9}$$

reaches a maximum different from zero then the control system described with equations (1) with optimal controller G^* is optimal in terms of signal class $R(F)$.

The above condition can be also noted for spectral transmittance

$$\inf_{-\infty < \omega < \infty} \left| \frac{1}{F(j\omega)} + G(j\omega) \right| = r(G) \tag{10}$$

Geometrically, this means that the smallest distance between the spectrum of the operation $1/F$ and the spectrum of operation –G equals the constant, $r(G)$. We try to select controller G so that constant $r(G)$ will be as big as possible and therefore, the signal norm – as small as possible.

The robust controller selection methodology compliant with the above considerations requires advanced mathematical knowledge in functional analysis from the control system designer. However, evolutionary algorithms can be a perfect tool for optimization of control systems based on robust controller. The application of evolutionary methods relieves us from the requirement of being familiar with functional analysis.

2. Example of a robust controller selection

A ship was assumed as the controlled object. The block diagram for the ship route control system is illustrated in fig. 2.

The following of the robust controller for the autopilot was assumed:

$$G(s) = \frac{a_1 s^2 + b_1 s + c_1}{a_2 s^2 + b_2 s + c_2} \tag{11}$$

The coefficients of the equations in the numerator and the denominator are sought for the optimal form of the robust controller. During the selection of a robust controller, it has to be taken into consideration that, according to the geometrical interpretation of an optimal controller, the sets limited with Nyquist curves for operations $1/F(j\omega)$ and $-G(j\omega)$ must be disjoint. The separation of those curves guarantees the stability of the feedback control system. The change of velocity and the change of the rudder angle lead to the change of the linear form of

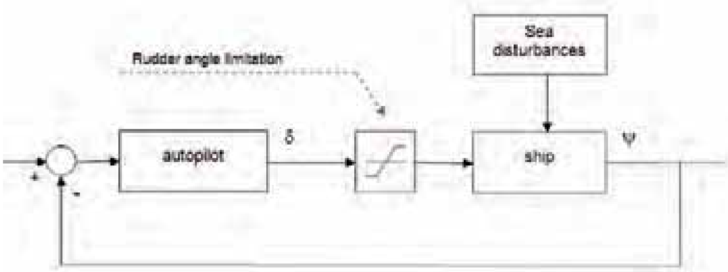

Fig. 2. Ship route control system with autopilot in the form of a robust controller

Rys. 2. System śledzenia trajektorii statku z autopilotem w postaci regulatora odpornego

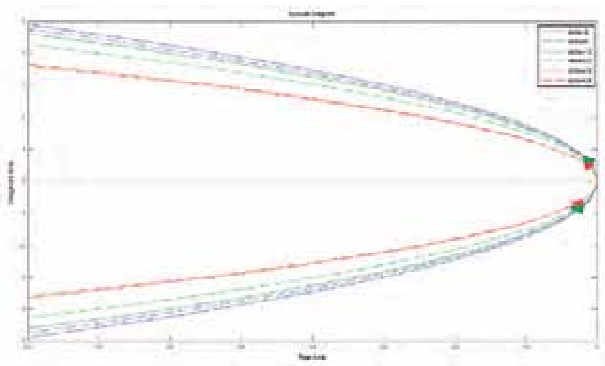

Fig. 3. The family of Nyquist characteristics for the ship, depending on the rudder angle δ

Rys. 3. Rodzina krzywych Nyquista w zależności od kąta wychylenia steru δ

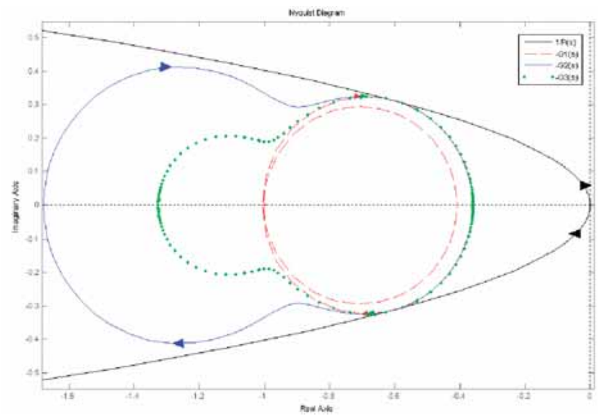

Fig. 4. Nyquist curves for controllers described with equations (5, 6) and for the ship

Rys. 4. Krzywe Nyquista dla regulatorów opisanych równaniami (5, 6) oraz statku

Nomoto's model of the ship as the controlled object. The selection of the robust controller requires application of a form of the object, whose frequency spectrum provides stability of all other possible forms of the object. In case of a ship this condition, according to fig. 3, is met by the form of the object described with the equation for the largest rudder angle ($\delta = 35°$).

During the computation, the minimization of the operation norm $||F(j\omega)^{-1} + G(j\omega)||$ in the frequency range of 0–0.16 rad/s is considered. There is no ground to consider higher frequencies due to the limitation of the operation speed of the rudder machine and the inertness of the ship. The optimization of the robust controller equation coefficients was performed with evolutionary algorithms. As the result, three various robust controller transmittances were obtained, but the forms of the norms for the frequency range in question are almost homogenous.

Fig. 4 illustrates Nyquist frequency characteristics for obtained controllers and operation $1/F(j\omega)$. It can be seen that the sets limited with the operation inverse to the ship model and controllers are disjoint, and the control system with those controllers shall be stable. Spectral characteristics of the controllers overlap in low frequency ranges.

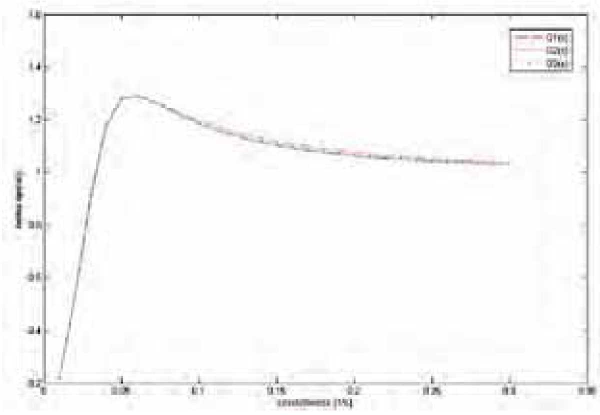

Fig. 5. Norm value for operation given with equation 10 for obtained controllers

Rys. 5. Wartości norm dla operacji opisanych równaniem (10) dla otrzymanych regulatorów

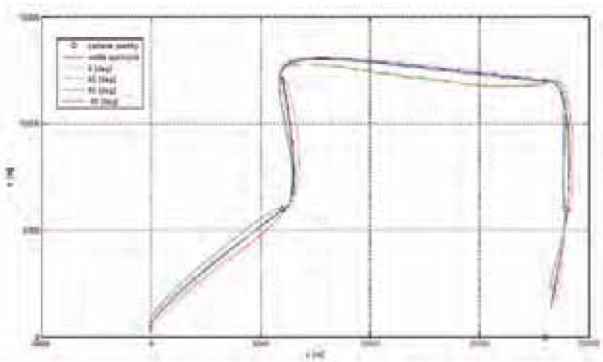

Fig. 6. Tracing the ship route at wind speed of 6 °B and various wind directions

Rys. 6. Symulacje śledzenia trajektorii statku dla prędkości wiatru 6 °B oraz różnych kierunków wiatru

Fig. 5 illustrates the norm of operations given with equation (10) for the obtained controllers $G1$, $G2$, and $G3$. The chart demonstrates that the norms for the controllers in question almost overlap.

Fig. 6 presents simulations of the ship travel towards consecutive set points, with wind speed of 6 °B and various wind directions while tracing the ship route with $G3$ controller

The traces of the routes covered by the ship, presented in fig. 6 are very similar or partly overlapping.

Bibliography

1. Francis B.A., *A Course in H_∞ Control Theory*, Springer-Verlag, Berlin 1987.
2. Nikończuk P., Łozowicki A., *Evolutionary algorithms application in a ship autopilot system with optimal controller.* 7th IFAC Conference on Manoeuvring and Control of Marine Craft Lisbon, Portugal 2006.
3. Nikończuk P., *Evolutionary algorithms application for optimal controller design*, "Polish Journal of Enviromental Studies", Vol. 17, No. 4C, 2008, 88–90.
4. Nikończuk P., Królikowski T., *Ewolucyjne metody projektowania regulatorów odpornych*, „Pomiary Automatyka Kontrola", Nr 4/2010, Vol. 56, 297–300.

Techniki ewolucyjne doboru regulatorów odpornych

Streszczenie: Matematyczne metody doboru współczynników regulatora odpornego w przestrzeniach H_∞ są bardzo skomplikowane. Projektant układu regulacji musi wykazywać się znajomością technik analizy funkcjonalnej. Do rozwiązywania problemów optymalizacji tego rodzaju doskonale nadają się algorytmy ewolucyjne. W artykule przedstawiono metodę oraz wyniki symulacji podczas doboru współczynników równania regulatora odpornego. Do doboru użyte są tylko dwa kryteria: sprawdzenie stabilności i zależność geometryczna – minimalizacja największej odległości między krzywymi Nyquista operacji $G(j\omega)$ i $1/F(j\omega)$, gdzie $G(j\omega)$ i $F(j\omega)$ są transmitancjami regulatora oraz obiektu regulacji w układzie sprzężenia zwrotnego.

Słowa kluczowe: algorytmy genetyczne, sterowanie odporne

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