Mathematical analysis of tubular linear motor

Krzysztof Falkowski, Maciej Henzel, Paulina Mazurek

Faculty of Mechatronics and Aerospace, Military University of Technology, Poland

Abstract: The paper deals with mathematical analysis of linear electric motor. There are characterized modern trends in aviation, especially in aircraft actuator field. There are also described operation principles of linear motor and transformation from cylindrical rotary motor to tubular linear motor. There are presented motion equation of linear motor and formula derivation of mutual inductances matrix. In three phase power supply mutual inductances are depend on slider linear movement.

Keywords: actuator, linear electric motor, More Electric Aircraft

Nowadays in aviation trends occur explicit tends to use electric equipment instead of device powered by pneumatic and hydraulic systems. This concept are named “More Electric Aircraft” and give a wide range of opportunity to increase device reliability, vulnerability, maintainability and flight safety. Thanks to this concept new aircraft produce less noise and pollution. Their weight, fuel consumption and utilization costs are reduced.

1. Introduction

In the last years, the aircraft systems are changed by the equivalent electrical systems, e.g. pneumatic anti-icing system are eliminated by electric heating mats, hydraulic actuators are changed by electric servo-actuators, mechanical control systems are evaluated to fly-by-wire systems. This on-board revolution are named “More Electric Aircraft” (MEA) technology. Advantages of this solution were integration of distribution system, energy storage, actuators and control systems. The MEA technology allow to get greater precision, rapidity and reliability of systems. This idea have an effect on increase of maintenance susceptibility, ensure systems flexibility during modification and reduce operation costs and system weight.

The More Electric Aircraft concept are used in passenger aircraft Boeing 787, Airbus A380, multipurpose fighter F-35 and in unmanned aircraft Predator and Global Hawk. There are applied e.g. anti-icing systems, hydraulic systems, environmental control systems, aircraft engines, electrical power and actuation systems. [4, 8]

Actuator is a component of control system, which converse control signal to physical process, e.g. movement, rotation, moment or force. In that system input signal are generated by flight control computer. Whereas output signals are movement motion signals of aircraft surfaces (flaps, rudders, elevator, ailerons). On aircraft are used electric, pneumatic and hydraulic actuation systems.

Electric actuators ensure operating flexibility of aircraft systems by reduction of hydraulic component, limitation of spare parts and tools, minimalize of diagnostic mistakes by built-in test function. Electric data transfer is more practical and ensure flexibility in case of modification [8].

Main advantage of electric actuator is reduction of aircraft operating costs (reduce fuel costs by minimization aircraft weight and minimalize number of maintenance services).

Linear motors are electro-mechanic converters, which converse received electric power to mechanical energy. This is alternative solution for hydraulic and pneumatic drives. Linear motor output is linear movement. This system does not require gear, couplings etc. It have better dynamic parameters (acceleration, velocity, breaking) and operating reliability than rotary drives. So, the structure of linear motor is simply. That motor consists of electric stator and slider with permanent magnets. Linear actuator are modern solution of actuation systems components. They allowed to quick movement and characterize operating precision. In most solutions are used air bearings, which eliminate friction forces.

2. Mathematical model ofaircraft actuator with linear electric drive

As an example of linear electric drive will be presented tubular linear electric motor

2.1. Tubular linear motor

Linear electric motor is an electrical machine, which converts electric energy to mechanic energy of progressive movement without any additional elements (gears, couplings, etc.). In turn, tubular motor mostly has cylindrical active surfaces, which primary winding coils are located in perpendicular plane to movement direction. [1]

Flat linear motor arise through cross-cut rotary electric motor and flat drop-down packets (active flat surfaces are separate by flat air gap – fig. 1a–b). The cross-cut is made by semi-plane limited by motor pivot *Or*. While tubular motor arise through wind up flat motor around perpendicular pivot *Ot* to rotor motor pivot (tangential coordinate of rotary motor become linear coordinate of tubular motor oriented along him pivot, linear coordinate oriented along rotary motor pivot become tangential coordinate of tubular motor – fig. 1b–c). Air gap of tubular motor is also cylindrical, but moving member move along pivot (linear movement). [1]

a) b) c)

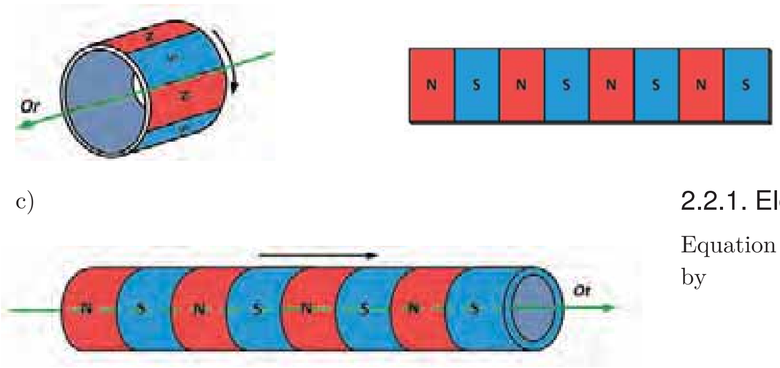

Fig. 1. Transformation from cylindrical motor to tubular motor; a) cylindrical motor, b) linear flat motor, c) tubular linear motor

Rys. 1. Ewolucja silnika cylindrycznego w silnik tubowy; a) silnik cylindryczny, b) silnik liniowy płaski, c) silnik liniowy tubowy

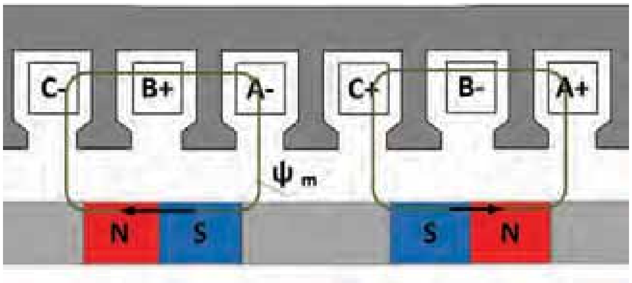

Fig. 2. Magnetic flux from permanent magnets in tubular linear motor

Rys. 2. Strumień magnetyczny od magnesów trwałych w tubowym silniku liniowym

Main components of linear motor are inductor and slider (equivalent of stator and rotor in rotary machine). Principle of operation that kind machines based on influence between magnetic field generated by inductor winding and magnetic field derive from magnets located on slider. Range of slider movement is limited by race length.

2.2. Mathematical analysis of tubular linear motor

Electric equation and motion equation of linear motor, can be express by Eq. (1) and Eq. (2).

$$\underline{u}(t) = \underline{R}\underline{i}(t) + \frac{d}{dt}\underline{\psi} \tag{1}$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx + F_{zew}sign(\frac{dx}{dt}) = F \tag{2}$$

where:

$\underline{\psi}(t)$ – column matrix of linkage fluxes on m_u-phases,
$\underline{R}$ – quadratic diagonal matrix of resistances (rank of matrix is m_u),
$\underline{u}(t)$ – column matrix of phase voltages with m_u-rows,
$\underline{i}(t)$ – column matrix of phase currents with m_u-rows,
m – mass of mobile elements,
m_u – number of phases,
b – friction factor,
k – spring rate,
F – electromagnetic force,
F_{zew} – external force,
d^2x/dt^2 – acceleration,
dx/dt – linear velocity,
x – linear movement.

2.2.1. Electric equation

Equation (1) for the three phase system can be expressed by

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} \tag{3}$$

where:

R_j – resistance of j-coil.

Whereas linkage magnetic flux on a, b and c phase can be presented by [7]

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \end{bmatrix} = \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \psi_{ma} \\ \psi_{mb} \\ \psi_{mc} \end{bmatrix} \tag{4}$$

where:

L_i – self-inductance of i-coil,
M_{ij} –mutual inductance of j-coil located in magnetic flux generated by k-coil of electric circuit.

On equation (4) $\psi_{ma,}$ ψ_{mb} and ψ_{mc} represents linkage magnetic fluxes from permanent magnets [6]

$$\begin{bmatrix} \psi_{ma} \\ \psi_{mb} \\ \psi_{mc} \end{bmatrix} = \psi_m \begin{bmatrix} \cos(\frac{\pi}{\tau}x) \\ \cos(\frac{\pi}{\tau}x - \frac{2\pi}{3}) \\ \cos(\frac{\pi}{\tau}x + \frac{2\pi}{3}) \end{bmatrix} \tag{5}$$

where:

τ – pole pitch.

Based on (4–5) equations linkage magnetic flux derivative can be expressed by

$$\begin{bmatrix} \frac{d\psi_a}{dt} \\ \frac{d\psi_b}{dt} \\ \frac{d\psi_c}{dt} \end{bmatrix} = \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{bmatrix} \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix} - \frac{\pi}{\tau}\frac{dx}{dt}\psi_m \begin{bmatrix} \sin(\frac{\pi}{\tau}x) \\ \sin(\frac{\pi}{\tau}x - \frac{2\pi}{3}) \\ \sin(\frac{\pi}{\tau}x + \frac{2\pi}{3}) \end{bmatrix} \tag{6}$$

Finally equation (3) can be presented by

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{bmatrix} \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix} - \frac{\pi}{\tau}\frac{dx}{dt}\psi_m \begin{bmatrix} \sin(\frac{\pi}{\tau}x) \\ \sin(\frac{\pi}{\tau}x - \frac{2\pi}{3}) \\ \sin(\frac{\pi}{\tau}x + \frac{2\pi}{3}) \end{bmatrix} \tag{7}$$

Magnetic flux from permanent magnets ψ_m can be calculated by

$$\psi_m = \frac{l_m}{l_m+\delta} B_r S \tag{8}$$

where:

l_m – permanent magnets thickness,

δ –air gap length,

S –permanent magnet area,

B_r –remanent flux density of the permanent magnets (for neodymium magnets B_r=1.3 T).

Resistance of j-coil from equation (7) can be calculated by

$$R_j = \rho \frac{l}{s} \tag{9}$$

where:

ρ –coil material resistivity (for copper $1.78\cdot 10^{-8}$ Ωm),

l_z – length of j-coil,

s – wire section.

Self-inductance of j-coil from equation (7) can be calculated by

$$L_j = \frac{\partial \psi_j}{\partial i_j} \tag{10}$$

Whereas mutual inductance of j-coil located in magnetic flux generated by k-coil of electric circuit

$$M_{jk} = \frac{\partial \psi_j}{\partial i_j} \tag{11}$$

where:

ψ_{jk} – magnetic flux linkage from j-coil, induce by k-coil current.[3]

Magnetic flux of harmonic ψ_j induction associate with windings current i_j [5]

$$\psi_j = z_j \zeta_v l_j \int_{-\tau/2}^{\tau/2} B_p(x,t)dx = \frac{4\mu_0}{\pi}\frac{z_j^2}{p}\frac{\tau l_j}{\delta_i} i_j(t)(\frac{\zeta_v}{v})^2 \tag{12}$$

Whereas magnetic flux linkage ψ_{jk} associate with j-windings generate by k-windings influence

$$\psi_{jk} = z_j l_j \int_{-\tau/2}^{\tau/2} B_p(x,t)dx = z_j \frac{4\mu_0}{\pi}\frac{z_k}{2p}\frac{\tau l_j}{\delta_i} i_k(t) \bullet \tag{13}$$

$$\bullet \int_{-\tau/2}^{\tau/2} \sum_v \left\{ \sigma \frac{\zeta_v^2}{v} \cos v\left[\frac{\pi}{\tau}x + (j-k)\frac{2\pi}{m_u}\right]\right\} dx$$

where:

$\delta_i = \delta k_c k_{ns}$,

$\sigma = \sin v(\pi/2)$,

l_j – ideal length of slot,

μ_0 – vacuum magnetic permeability,

p – number of poles,

k_c – Carter`s coefficient (increase air gap coefficient),

k_{ns} –saturation coefficient,

v – number of pole harmonic,

ζ_v– windings coefficient,

x – coordinates.

Where after take into account only fundamental component v=1

$$L_j = \frac{4\mu_0}{\pi}\frac{z_j^2}{p}\frac{\tau l_j}{\delta_i}\zeta_v^2 \tag{14}$$

$$M_{jk} = \frac{4\mu_0}{\pi}\frac{z_k z_j}{p}\frac{\tau l_j}{\delta_i}\zeta_v^2 \cos(j-k)\frac{2\pi}{m_u} \tag{15}$$

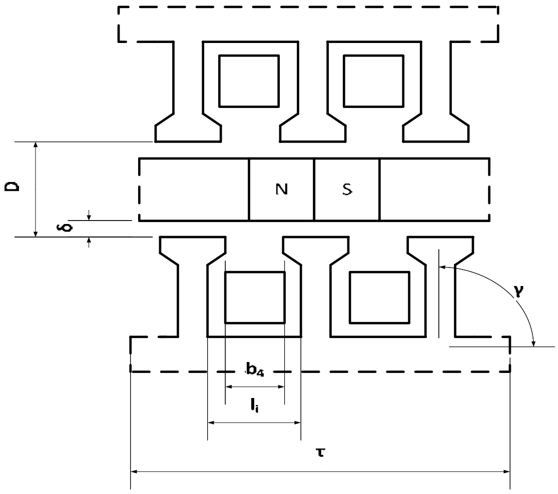

Fig. 3. Scheme of geometric parameters designations of motor

Rys. 3. Schemat oznaczeń parametrów geometrycznych silnika

2.2.2. Motion equation of linear tubular motor

Electromagnetic force F from equation (2)for three phase system, which act on slider in direction x is described by [7]:

$$F = \frac{\partial}{\partial x}\left(\frac{1}{2}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}^T \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}^T \begin{bmatrix} \psi_{ma} \\ \psi_{mb} \\ \psi_{mc} \end{bmatrix}\right) =$$

$$\frac{\pi}{\tau}\psi_m \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}\begin{bmatrix} -\sin\frac{\pi}{\tau}x \\ -\sin(\frac{\pi}{\tau} - \frac{2\pi}{3}) \\ -\sin(\frac{\pi}{\tau} + \frac{2\pi}{3}) \end{bmatrix} \tag{16}$$

Equation (2) for the three phase system allowing of equation (16) can be express by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx + F_{zew} sign(\frac{dx}{dt}) = \frac{\pi}{\tau}\psi_m \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}\begin{bmatrix} -\sin\frac{\pi}{\tau}x \\ -\sin(\frac{\pi}{\tau}x - \frac{2\pi}{3}) \\ -\sin(\frac{\pi}{\tau}x + \frac{2\pi}{3}) \end{bmatrix} \tag{17}$$

2.2.3. Coefficient evaluate

Coefficient from chapter 2.2. can be calculate:

– friction factor b:

$$b = k_\mu \frac{S_t}{\delta} \tag{18}$$

where:

k_μ– coefficient of internal friction (for air k_μ = 0,000018 Pa · s),

S_t– friction surface,

δ – air gap between parallel friction surfaces, $\delta << (S_t)^{0,5}$;

– windings coefficient ζ_v ,

$$\zeta_v = \zeta_{qv} \zeta_{sv} \zeta_{skv}; \tag{19}$$

– group coefficient ζ_{qv},

$$\zeta_{qv} = \frac{\sin\left(v \frac{q}{2} \frac{\pi}{Q}\right)}{q \sin\left(v \frac{1}{2} \frac{\pi}{Q}\right)}; \tag{20}$$

where:
q – number of slots for pole and phase,
Q – number of slots for phase (in three phase windings $Q = 2q$),

– pitch factor ζ_{sv},

$$\zeta_{sv} = \sin\left(v \frac{\pi}{2} \frac{y}{\tau}\right); \tag{21}$$

where:
y – coil span calculated in slots

– bevel factor ζ_{skv},

$$\zeta_{skv} = \frac{\sin\left(v \frac{\gamma}{2}\right)}{v \frac{\gamma}{2}}; \tag{22}$$

where:
γ – angle of slot bevel relative rotor generating line.

Tab. 1. Parameters of designed tubular linear motor
Tab. 1. Parametry zaprojektowanego liniowego silnika tubowego

Parameter	Value of parameter	Unit
Number of phases	3	-
Number of coils per phases	240	-
Slider weight	1.170	kg
Saturation coefficient	1,1	-
Carter's coefficient	1.075	-
Ideal length of slot	295.16	mm
Slot opening	4	mm
Number of poles	8	-
Number of slots	12	-
Inside diameter of stator	25.5	mm
Friction coefficient	0.01	-
Air gap length	0.5	mm
Slots angle	90	°
Length of coils	33600	mm
Windings coefficient	0.76	
Permanent magnet area	602.88	mm^2
Permanent magnets thickness	6	mm
Pole pitch	33	mm
Additional waste coefficient	2.6	-
Conductor conductivity	59770000	S/m
Wire section	0.785	mm^2
Winding temperature	40	°C

On the average Carter's coefficient is situated in range from 1.05 to 1.1 for half-closed slots and from 1.2 to 1.3 for opened slots. Whereas saturation coefficient average from 1.05 to 1.6.

Parameters of designed tubular linear motor were presented in tab. 1, whereas calculated parameters of tubular linear motor were shown in tab. 2.

Tab. 2. Calculated parameters of designed tubular linear motor
Tab. 2. Obliczone parametry zaprojektowanego liniowego silnika tubowego

Parameter	Value of parameter	Unit
$R_a=R_b=R_c$	0.762	Ω
$L_a=L_b=L_c$	0.0906	H
$M_{ab}= M_{ba}=M_{cb}=M_{bc}=M_{ac}=M_{ca}$	0.0452	H
Ψ_m	$0.868 \cdot 10^{-3}$	$T{*}m^2$

3. Summary

Paper presents modern trends in aircraft control system, especially in electric actuator. Tubular motor is an example of solution developed according to More Electric Aircraft concept.

There are characterized electric actuators, linear motor and operation principle of tubular linear motor. There are also presented mathematical model of linear motor. That model was created for first pole harmonic with assumption shaft length is infinite (omission of marginal phenomena) and air gap length is uniform.

Presented mathematical model is base to create simulation model, which can be verify in experimental research.

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Analiza matematyczna tubowego liniowego silnika elektrycznego

Streszczenie: W artykule przedstawiono analizę matematyczną liniowego silnika elektrycznego. Scharakteryzowano tutaj nowe trendy rozwoju lotniczych układów wykonawczych. Opisano również zasadę działania silnika liniowego. W artykule zaprezentowano równania ruchu silnika liniowego oraz wyprowadzenie macierzy indukcyjności własnych jego uzwojeń. W przypadku zasilania prądem trójfazowym indukcyjności wzajemne zależą od ruchu biegnika, dlatego przedstawiona analiza jest istotna.

Słowa kluczowe: lotniczy układ wykonawczy, liniowy silnik elektryczny, *More Electric Aircrat*

Krzysztof Falkowski, PhD

Krzysztof Falkowski graduated Military University of Technology. He received PhD title in 1999. He does research about magnetic suspensions, magnetic bearings and bearingless electric motors. He is author or co-author of many articles about magnetic levitation phenomena. He is organizer of Magnetic Suspension Workroom of Aircraft Engines Laboratory in Military University of Technology.
e-mail: krzysztof.falkowski@wat.edu.pl

Maciej Henzel, PhD

Maciej Henzel graduated of Military University of Technology. He received PhD title in mechanics discipline and control systems specialization in 2004. He works in Military University of Technology since 1998. He does research aircraft control and actuation systems, measurement systems and bearingless machines. He is author or co-author of many articles about new trends in on-board systems, modern control methods and bearingless drives.
e-mail: maciej.henzel@wat.edu.pl

Paulina Mazurek, MSc

Paulina Mazurek graduated of Military University of Technology. She has worked in Military University of Technology since 2011. Her areas of interest are aircraft control and actuation system and bearingless machines. She is author or co-author of many articles about new trends in on-board systems, modern control methods and bearingless solutions.
e-mail: paulina.mazurek@wat.edu.pl